EXPERIMENTAL STUDY OF MULTI-WAVE RESONANT INTERACTIONS IN A NON-Self Similar Boundary Layer ON AN AIRFOIL

I. B. De Paula¹, D. Sartorious¹, W. Würz¹, E. Krämer¹, Y. S. Kachanov²

¹Institut für Aerodynamik und Gasdynamik, Universität Stuttgart
70550, Stuttgart, Germany
²Institute of Theoretical and Applied Mechanics, RAS-SB,
630090 Novosibirsk, Russia

Introduction

For the design of subsonic natural laminar flow (NLF) airfoils, usually codes based on boundary layer Linear Stability Theory (LST) are used for prediction of the transition location. The accuracy of such codes decreases as the extension of non-linear region increase in comparison to the linear part. For very short non linear regions a rough estimation of the transition location can be obtained. Detailed optimizations of the boundary layer parameters with respect to a long laminar run, now take the advantage of successive (linear) amplification and damping of Tollmien-Schlichting (T-S) waves in a way that the resulting amplitudes just remain below a critical threshold. In this case the onset of non linear interactions is of significant importance to extend the laminar run.

At weakly-nonlinear stages of transition resonant interactions between different Tollmien-Schlichting modes often play a dominant role. The classical resonant scenario was first given by Craik [1]. In that case the resonance consisted in the interaction between a single 2-D fundamental wave with a pair of 3-D subharmonics with half of the fundamental frequency. The classical scenario was further extended in [2] and [3]. Those works have shown that the resonance is not only possible for the classical tuned case, where frequency, spanwise wavenumber and phase fit but also for detuned resonances.

A study of these non-classical regimes is now extending to the practical case of a boundary layer on an airfoil. According to [4], the influence of the pressure gradient on the weakly nonlinear interactions seems to be mainly a result of changes of the linear stability characteristics of the base flow. The current work is an extension of that previous investigation for the non self-similar boundary layer case. In the present case, the focus is on the investigation of interactions between several modes under such conditions. The current results are compared with the ones obtained in [5].

Experimental Investigation

The experiments were conducted in the Laminar Wind Tunnel of the Institute for Aerodynamics and Gasdynamics (IAG) of Stuttgart University. The turbulence level of the flow is below 0.02% in the frequency range of 20-5000 Hz for the velocity of the current measurements ($U_\infty =18$m/s). The boundary layer measurements were performed on the lower surface of the WW03BL106 airfoil section ([5]). This profile was specially designed to give a constant threshold of a $n_{factor}=\ln(A(f)/A_0(f))=6$ at an angle of attack of $-2$ degree and at Reynolds number of $0.7 \times 10^6$.

The experiments were carried out at controlled disturbance conditions. The T-S waves were excited by a slit source, which was mounted flush with the surface at $s/s_{max}$ equals to 0.13, where $s$ is the position measured along the surface and $s_{max}=0.604$ is the corresponding reference length. The slit aperture is 0.2 mm wide and extends 290 mm in the spanwise direction. There are 116 equally spaced pneumatic tubes below the slit connected to 32 loudspeakers, which are driven by power amplifiers and a 16 channel digital signal generator. This device enables the generation of

disturbances with different frequencies and spanwise wavenumbers. Phase-locked hot-wire measurements were performed downstream of the slit. The phase locking was set with respect to the disturbance generator. Sets of spanwise and wall-normal profiles were obtained at several downstream positions. All necessary base flow parameters were measured, including stability characteristics for 2-D and 3-D T-S waves in a range of frequencies of the current experiments (approximately from 200 to 700 Hz).

The Hartree parameter ($\beta_H$), given in equation 1, was used to characterize the flow conditions along the streamwise region of measurements. The calculated (Xfoil) Hartree parameters for the conditions of the current experiments are shown in figure 1. This figure also indicates the value of $\beta_H$ (-0.115) for previous investigation in self similar boundary layers [4]. The test set-up presents a higher adverse pressure gradient at the position of the source ($\Delta s=0$). In this region, the Hartree parameter reaches a minimum value of -0.15. Note that the limit for separation of the laminar boundary layer is -0.199. As the flow develops downstream the base flow tends asymptotically to a Blasius boundary layer.

\[
\beta_H = \frac{2 \xi}{U_e(x)} \frac{dU_e(x)}{d\xi}, \quad \xi = \int_0^x U_e(x) dx \tag{1}
\]

The measurements concentrated on the range of Tollmien-Schlichting waves with frequencies close to maximum amplification due to primary instability. The streamwise amplification curves of two single 2-D Tollmien-Schlichting waves are shown in figure 2. The theoretical amplification curves obtained by means of Linear–Stability-Theory (LST) calculations are included for comparison. The numerical code used for these calculations is described in [7]. The LST results were obtained for the non-self similar boundary layer of the airfoil at an incidence angle of $-2^\circ$. Usually, amplification rates are much more sensitive to deviations in the adjustment of the angle of attack than pressure distributions and mean flow parameters. Therefore, the good agreement between theory and experiments indicates that the incidence angle was accurately adjusted.

The characteristics of the waves introduced artificially in the flow by the source were also checked. The procedure for the adjustment of the waves is given in [5]. The wall normal profiles and the spanwise distributions of the T-S waves amplitudes obtained from the measurements at 80mm downstream the source are shown in figures 3 and 4. Again a good agreement with theory was obtained.
Multi-wave resonance

The resonance in the multi-wave regime can be suggested as a step towards the investigation of the natural transition. According to [5], in this regime, the interaction of the modes can promote the amplification of frequency detuned sub-harmonic modes. The influence of the frequency detuning on the resonant interactions of a single triad, composed of one fundamental 2-D and a pair of sub-harmonic oblique waves, was extensively investigated in [4]. It is observed in [4] and [5] that frequency detunings of the sub-harmonic waves induce modulation on sub-harmonic modes. The observed period of modulation was equal to $1/\Delta f$, where $\Delta f$ is the frequency detuning. In the regime of multi-wave resonance, the modulation of the fundamental 2-D T-S waves is already present in the signal. The modulation period is given by $1/(|f_1 - f_2|)$, where $f_1$ and $f_2$ are the frequencies of the fundamental waves. The investigation of the evolution of modulated waves was extensively studied in [8] for wave packets and wave trains. The wave modulation adopted in that work was based on window functions, thus it is difficult to extract from it, the influence of the modulation on the amplification rates of the waves. According to [8] an increasing of the modulation of the packets induces earlier development of nonlinearities in the flow. However, those results do not clarify if this effect is induced by higher initial seeds for nonlinear interactions or/and by higher growth rates due to the presence of several fundamental waves. The comparison of the growth rates obtained in the single resonant regime and the ones observed when several fundamental waves are present in the flow, would provide such information. However, prior to the comparisons it is necessary to ensure that the conditions for resonance are strictly the same in both cases.

It is well known that the amplitude of the fundamental waves influences significantly role on the amplification rates due to secondary instability mechanisms. Thus, for comparisons between single wave and multi-wave resonant regimes it is necessary to establish an equivalence criterion for amplitude of the waves for the regimes of single and multi wave interactions. In the current work, this criterion is chosen based on the results presented in [4]. The amplitude of the modulated sub-harmonic waves was calculated by the single relation: $A_{\text{eff}} = A_{f_1} + A_{f_2}$, where $A_{\text{eff}}$ is the effective amplitude and $A_{f_1}$ and $A_{f_2}$ are the amplitudes of the two modes which compose the modulated sub-harmonic.
In order to clarify how the addition of one fundamental wave can affect the effective resonance frequencies some experiments were performed with single detuned sub-harmonic resonant triads [regime (i)]. These cases were taken as a reference. Afterwards, some experiments in multi-wave regime (ii) were performed and the results were compared.

The spectra of velocity fluctuations in the regimes (i) and (ii) are shown respectively in figures 5 and 6. Each spectrum is shifted in the amplitude scale by 1.5 decades. The time series of the velocity fluctuations were acquired at a distance from the wall that corresponds to the maximum in the sub-harmonic wave amplitude profile. For instance, at $\Delta s = 20\text{mm}$ this corresponds approximately to the boundary layer displacement thickness ($y/\delta^* \approx 1$). The spanwise position was fixed at the peak of the oblique sub-harmonic waves. The amplitude of single and effective fundamental waves in all experiments were equal to 0.06% of $U_e$ at $\Delta s = 20\text{mm}$. The spanwise wavelength of the excited sub-harmonic waves was 15mm. This spanwise periodicity was set based on previous measurements [5] performed at the same experimental conditions of the current work. According to [5], for such conditions, these sub-harmonic 3-D waves were the most amplified ones by the secondary instability.

Initially the spectral content of the signals shown in figure 5 and 6 are mainly composed of the excited modes. As the waves propagate in the streamwise direction the energy in other frequencies became quite evident. At the last streamwise station the energy of symmetric modes with a frequency spacing of $df = f_{\text{SUB}} - f_{\text{Detuned}}$ is remarkable. The frequency of the excited waves is marked in the axes of the figures 5 and 6. According to [2], the symmetric modes in the regime (i) appear due to a beating like resonance which provides the growth of a modulated sub-harmonic mode. In these cases, the carrier frequency (effective frequency) of this modulated signal is equal to the exact sub-harmonic frequency ($1/2$ fundamental frequency). This effective frequency, according to [4] is given by $f_{\text{Eff}} = (f_{\text{Sym1}} + f_{\text{Sym2}})/2$.

Figure 5: Frequency spectra of velocity fluctuations obtained for single detuned regime of sub-harmonic resonance (i).

Figure 6: Frequency spectra of velocity fluctuations obtained for multiple-wave regime of resonance (ii). Case with one quasi sub-harmonic mode.

The behavior of this beating was also investigated for the regime where two fundamental waves were excited in the flow [(ii)]. In figures 5 and 6 the frequency of the sub-harmonics was 1% detuned with respect to the exact sub-harmonic of one fundamental wave. This method of small detuning was used in [5] to reduce the influence of the initial phase of the waves on the resonance of the waves.
Spectra of figure 6 [regime (ii)] shows some additional symmetric modes in comparison with the case of single detuned regime (i). Moreover, the central frequency of the symmetric modes is not clear.

The amplifications of sub-harmonic modes obtained in the regimes (i) and (ii) were compared in order to observe how the amplification rates are sensitive to the addition more fundamental modes. For this comparison, one of fundamental frequencies was set to be equal in the two regimes (about 610Hz). In the multi-wave regime just an additional fundamental wave (about 516Hz) was added. In the regime (ii), the initial amplitude of both fundamental waves had to be reduced by a factor of 2, in order to save the condition of same amplitude in the two regimes. Thus, the effective amplitude in the multi-wave regime was roughly close to the amplitude of the fundamental wave in regime (i), (figure 7). Figure 7 shows that the amplitudes of the effective fundamental modes do not fit accurately to the single wave due to differences in the amplification rates of the fundamental mode of lower frequency. Nevertheless, the result shows qualitatively, for this case, that no great difference in the amplification rates of the effective sub-harmonic modes occur when the effective amplitude of the fundamental waves are close. The modes used for the calculation of the effective sub-harmonic amplitude are indicated with arrows in figures 5 and 6.

![Figure 7: Amplification of effective fundamental and sub-harmonic modes in regimes (i) and (ii).](image)

For a better evaluation of the influence of the addition of one fundamental mode on the growth rates of the effective sub-harmonic waves, it was selected a pair of fundamental waves with an effective frequency ($f_{\text{eff}}=f_1+f_2$) equals to the frequency of the fundamental wave in the regime (i). In this way, the linear growth rates of the fundamental waves would be closer in the two regimes. Thus, the energy involved in non-linear interactions would be also close in the two regimes. In figure 8, the amplitude of the effective fundamental and sub-harmonic waves are compared with the ones obtained in regime (i). As can be seen in the figure 8, with these conditions the growth rates are in very good agreement. Just at the last streamwise stations, the growth rates show some deviations due to the influence of the second branch of the primary instability. In the case of the modulated fundamental wave, the mode with lower frequency continues to grow while the mode at the effective frequency is already being damped. This difference of energy is also seen in the growth rates of the effective sub-harmonic modes.
The comparison of the growth rates of the effective modes suggest, for the investigated regime, that if the energy of the effective fundamental modes is similar in single and multi-wave regimes, then the resonant interactions might be also comparable.

In the natural transition a broad band of modes are present. In this regime the effective energy would be high. Thus, the influence of the energy increase due to the addition of one fundamental wave was experimentally investigated. In this case, the effective amplitude of the fundamental mode in the regime (ii) was higher than in the regime (i), despite the individual amplitudes of the excited modes in regime (ii) were slightly lower. Spanwise scans at a constant wall normal position were performed in such measurements. The wall normal position chosen for the scans was that one close to the peaks of the most amplified sub-harmonic wave. For these experiments a controlled background noise was imposed to the generated wave. This background noise was generated as suggested in [5]. The evolution of the disturbances at the condition of regime (i) is observed in figure 9. In this case, the spectral distribution at short and moderate distances from the source ($\Delta s=40$ and 80mm) present more energy around two-dimensional modes. This suggests an evolution driven by primary instability growth rates. Further downstream this pattern is modified and is evident the energy of oblique modes in a broadband of sub-harmonic frequencies. Oblique sub-harmonic modes with spanwise wave numbers around $0.1\text{mm}^{-1}$ presented a high energy level, at the last station ($\Delta s=120\text{mm}$). At this station, the growth rates of the fluctuations seem to be dominated by secondary instability mechanisms.
At the stations close to the source, the spectral distribution of figure 10 shows a behavior similar to the observed at regime (i). In this region, the non-linear effects were not evident. At the last measurement station, some differences in the total energy of the two spectral distributions can be seen. In the regime (ii), with higher effective amplitude of the waves, the total energy on the oblique modes was also higher. This suggests that differences on effective energy can modify the intensity of the non-linear interaction in the boundary layer.
Final Remarks

The resonance of T-S waves in the presence of multi fundamental modes was experimentally investigated in a non self similar boundary layer. The results show an increasing number of detuned sub-harmonic waves, that are amplified downstream of the T-S source, in comparison to single resonant triads. However, the amplification rates of the effective sub-harmonic modes do not seems to be affected by the addition of a second fundamental wave. This was observed in the experiments when the effective amplitude of the fundamental waves was comparable to the amplitude of a single T-S wave. This means that two superimposed detuned sub-harmonic-type resonances do not impede each other but work “in parallel”. Their superposition is able to amplify quasi sub-harmonic modes as efficient as every of this resonances separately.

The interaction of the T-S waves with a controlled background noise was also investigated for regimes (i) and (ii). In these investigations the effective amplitude of the fundamental wave was higher in regime (ii). The spectral distributions of the disturbances, show no significant differences in stations were the effects of the primary instability are dominant. At stations far from source (∆s=120mm), the difference of the total energy of the spectral distributions became more evident. In the regime (ii) the flow presents slightly higher amplitude of oblique modes with spanwise wavelength close to Craik’s modes. In both cases, for the range of the parameters analyzed, the wavenumber of the most unstable oblique mode was not affected by the addition of a second fundamental mode. However, in this last case a broader band of amplified oblique modes was observed. It might be possible that, for different levels of effective energy some differences on the bandwidth of unstable oblique modes can be observed.

In summary, the results suggest that resonant interaction between modulated 2D TS-waves and weak (initially) broadband background 3D TS-waves seems to be a dominant weakly-nonlinear mechanism of “natural” laminar-turbulent transition in airfoil boundary layers, which has to be taken into account in advanced approaches to transition prediction and control.

Acknowledgement

This work was performed under grant of the Deutsche Forschungsgemeinschaft (Wu 265/3-1) and the Russian Foundation for Basic Research

REFERENCES