REGULAR AND SINGULAR COMPONENTS OF LOW AND HIGH-SPEED FLOWS

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Introduction. Closing the discussion on fluid mechanics at the Second School on Hydrophysics in 1986 Academician S.A. Khristianovich reminded problems of setting in operation of the first trans-sound velocity wind tunnel [1]. He noted that the problem of wall-induced disturbances was solved practically by suction of flow but not mathematically. The goal of the talk is to present model of formation of thin disturbances in flows basing on the fundamental fluid mechanics equations. The main feature of the model is description of low flow velocity analogues of shock waves.

Shock wave is one of the most important features of high-speed flows. They are distinguished out from other kinds of flows in the conventional fluid mechanics that are jets, wakes, waves and vortices. As physical properties of media (density, temperature, pressure and velocity) change greatly inside relatively narrow and extended domains, these waves affect drag on moving obstacles and impact negatively on the environment. Being a subject of high-speed flows, shock waves are studied experimentally at expensive and energy consuming experimental facilities. Mathematical description of the flow is constitutive and based on combined schemes operating with the Euler equation for the main flow and Navier-Stokes equation in high gradients domains. The fluid is assumed to be barotropic, and density is constant in an undisturbed space. Temperature or concentrations fields are calculated in approximations of “passive admixtures” which do not affect flow structure and dynamics.

But the density of real fluids (that are liquids and gases) depends on pressure, temperature, and concentration of dissolved, or suspended matter, and is distributed non-uniformly in the physical space. Fluids in the environment and in laboratory facilities are stably stratified continuously or discretely due to non-uniformity of temperature or concentrations of dissolved or suspended matters.

Stratification and general rotation, even very weak, lead to some new phenomena, which do not exist in a homogeneous fluid. Among them there are inertial and internal waves and so-called ‘large scale’ and ‘fine structures’ forming by sets of thin high-gradient interfaces. The interfaces exist for rather a long time with respect the specific diffusion time for their transverse length-scales. It is well known that stratification strongly affects flow separation and downstream wake structure. High gradient interfaces produce picturesque patterns of environmental flows and their laboratory models. For better understanding the flow past obstacles and in order to compare observations with theoretical solutions the stratified flows past obstacles of a simple or perfect shapes, such as strips and right circular cylinders have been investigated. Progress in analysis, computing and in laboratory technique gives enough room for scrutiny theoretical study of the flows basing on exact solutions of governing equations or their linearized residuals satisfying to the real boundary conditions.

The main goal of the paper is to illustrate results by numerical and schlieren visualizations of all components of continuously stratified flow patterns. Quantitative comparison of stratified flows singular components and shock waves in high-speed flows is given, too.


Individual physical properties of fluids are characterised by empiric equations of state, which expresses the density \( \rho = \rho(p, T, S_j(z)) \) as function of pressure \( p \), temperature \( T \) or concentration of dissolved or suspended matter \( S_j \). The undisturbed density profile of a stratified fluid \( \rho(z) = \rho_o \exp(-z/\Lambda) \) is characterized by buoyancy scale \( \Lambda = (d \ln \rho(z)/dz)^{-1} \), frequency
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\[ N = \sqrt{g/\Lambda} \] and period \( T_p = 2\pi / N \), which are supposed to be constant. Dynamics of fluids is described by the set of fundamental equations including empirical equation of state and differential equations of continuity by D'Alembert, transport of momentum by Navier-Stokes, heat by Fourier and a matter by Fick [2]

\[
\begin{align*}
\rho &= \rho(s, \rho, S), \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \\
\frac{\partial \left( \rho \mathbf{v}^i \right)}{\partial t} + \nabla \cdot \mathbf{v} \Pi^j = \rho g^i + 2\rho \varepsilon^{ijk} v_j \Omega_k \\
\frac{\partial T}{\partial t} + \nabla \cdot (T \mathbf{v}) - \kappa_T \Delta T &= 0 \\
\frac{\partial S}{\partial t} + \nabla \cdot (S \mathbf{v}) &= \kappa_s \Delta S
\end{align*}
\]

\[(1)\]

where \( \mathbf{v} \) is velocity, \( \Pi^j = \rho \mathbf{v}^i \mathbf{v}^j + p \delta^{ij} - \sigma^{ij} \) is tensor of density of momentum flux, \( \delta^{ij} \) is fundamental metric tensor, \( \sigma^{ij} \) is symmetric tensor of viscous stresses, \( \varepsilon^{ijk} \) is unite completely antisymmetric tensor, \( \Omega \) is vector of general angular rotation of medium, \( g \) is gravity acceleration, ускорения силы тяжести, \( \kappa_T \) and \( \kappa_s \) are temperature and salinity diffusion coefficients. Boundary conditions on solid surfaces \( \Sigma \) are no-slip for velocity and no-flux for temperature and matter

\[ \mathbf{v}|_{\Sigma} = (S \mathbf{v} - \kappa \nabla S) \cdot \mathbf{n}|_{\Sigma} = 0 \]

\[(2)\]

The set of fundamental equations (1) is universal, invariant with respect of temporal and spatial scales and describes both slow and fast flows of fluids. The high order set (1) is singular disturbed type as well as small coefficients \( \nu, \kappa_T, \kappa_s \) present in terms with the highest derivatives. General methods of analysis of such systems are still not developed. As the first step the classification of fluid flows components is given in linear approximation when the set (1) is reduced to the form [2]

\[
\begin{align*}
\frac{1}{c^2} \frac{\partial \vec{v}}{\partial t} - \frac{w g}{c^2} + \nabla \cdot \mathbf{v} + \kappa_s \Delta \vec{S} &= 0, \quad \frac{\partial \rho}{\partial t} - \frac{W}{\Lambda} + \nabla \cdot \rho \mathbf{v} = 0, \quad \frac{\partial \vec{S}}{\partial t} - \frac{W}{\Lambda} - \kappa_s \Delta \vec{S} = 0 \\
\frac{\partial u}{\partial t} &= -\frac{\partial \vec{p}}{\partial x} + 2\Omega \left( v \sin \varphi - \frac{1}{\sqrt{2}} \cos \varphi \right) + \nu \Delta u + \left( \mu + \frac{\nu}{3} \right) \frac{\partial}{\partial x} \nabla \cdot \mathbf{v} \\
\frac{\partial v}{\partial t} &= -\frac{\partial \vec{p}}{\partial y} + 2\Omega \left( \frac{1}{\sqrt{2}} \cos \varphi - u \sin \varphi \right) + \nu \Delta v + \left( \mu + \frac{\nu}{3} \right) \frac{\partial}{\partial y} \nabla \cdot \mathbf{v} \\
\frac{\partial w}{\partial t} &= -\frac{\partial \vec{p}}{\partial z} + 2\Omega (u-v) \cos \varphi + \nu \Delta w + \left( \mu + \frac{\nu}{3} \right) \frac{\partial}{\partial z} \nabla \cdot \mathbf{v} - \rho g
\end{align*}
\]

\[(3)\]

for flows on rotating self-gravity sphere, \( \varphi \) is latitude at the point of observations, \( \nu, \mu \) are first and second kinematic viscosities and \( c \) is sound velocity [3].

The particular solutions of system (3) describing motions, which are periodic in time with a fixed real frequency \( \omega \) and complex wave vector \( \mathbf{k} = (k_x, k_y, k_z) \) are specified in the form of harmonic waves \( \mathbf{v} = \mathbf{v}_0 \tau(r, t), \quad \vec{p} = p_0 \tau(r, t), \quad \vec{p} = \rho_0 \tau(r, t), \quad \tau(r, t) = \exp \left(i(k \mathbf{r} - \omega t)\right)\).
The small intensity approximation of motions implies that the condition \( |\mathbf{k} \cdot \mathbf{v}| \ll \omega \) is satisfied and the weak-stratification approximation holds when \( |\mathbf{k}| \Lambda >> 1 \). Substitution of the expressions into system (3) generates a system of linear algebraic equations in the amplitudes \( u, v, w, \rho \). The dispersion equation follows from the condition for the non-trivial solvability of this system, that is, of the equality of its determinant to zero.

\[
D_\kappa(k) \left\{ \omega D_\nu(k) \left[ \omega D_\nu(k) \left( \tilde{D}_\nu(k) - \tilde{N}^2 \right) - 2\sqrt{2}\omega \Omega g \left( k_y - k_z \right) \cos \varphi - iD_\nu \tilde{N}^2 \left( \mu + \nu/3 \right) k_z^2 \right] + + 4\nu \Omega^2 \left[ \tilde{N}^2 \sin^2 \varphi - \omega \left( D_\nu + i \left( \mu + \nu/3 \right) f^2(k) \right) \right] + + c^2 \left[ D_\nu(k) \left( \tilde{N}^2 k_z^2 - \omega k^2 D_\nu(k) \right) + 4\nu \Omega^2 f^2(k) \right] \right\} + \kappa c^2 k^2 \Lambda^{-1} \left[ \omega k_z D_\nu^2(k) - 4\nu \Omega^2 f(k) \sin \varphi - - iD_\nu(k) \left( gk_z^2 + \sqrt{2}\nu \Omega \left( k_y - k_z \right) \cos \varphi \right) \right] = 0
\]

(4)

here \( D_\kappa(k) = \omega + ik^2 \), \( D_\nu(k) = \omega + i\nu k^2 \), \( \tilde{D}_\nu(k) = \omega + i\nu k^2 \), \( \tilde{N} = 4\nu/3 + \mu \), \( k^2 = k_x^2 + k_y^2 + k_z^2 \), \( k_z^2 = k_y^2 + k_z^2 \), \( \tilde{N}^2 = \frac{g}{\Lambda} \), \( \tilde{N}^2 = \frac{\tilde{N}^2 - g^2}{c^2} \), \( f(k) = k_z \sin \varphi + \left( \left( k_x + k_y \right) \cos \varphi \right) \sqrt{2} \).

The dispersion equation (4) is an eight order polynomial in the components \( k_z, k_x, k_y \) and, in the general case, it has a set of permissible solutions \( k_z(k_x, k_y) \) to each of which its own characteristic type of periodic motion corresponds. It gives a relation between the three components of the wave vector at a specified frequency \( \omega \). The action of gravitational forces separates out one of the components of the wave vector and, here, the dependence \( k_z(k_x, k_y) \) is conventionally understood to be the solution of the dispersion equation. It should be especially noted that, in dispersion equation, the small coefficient \( \kappa \nu \sim 2 \) is present accompanying the leading coefficient, that is, with respect to the wave-number \( k \), the equation belongs to the class of singularly perturbed equations.

The complete solution of the linearized system of fundamental equations is sought in the form of a superposition of the particular harmonic solutions

\[
A = \sum_{j} a_j( k_x, k_y) \exp \left( i \left( k_{x_j} k_x + k_{y_j} k_y \right) z + k_{x_j} x + k_{y_j} y - \omega t \right) dk_x dk_y
\]

(5)

The symbols \( A \) and \( a_j \) refer to each physical quantity (velocity, pressure or density) being considered, and its spectral representation, where the coefficients \( k_{x_j} k_x + k_{y_j} k_y \) are determined from the boundary conditions. It is important to emphasize that the summation in equality (5) must be carried out over all roots of dispersion equation (4), to which physically realizable solutions correspond. The set of such solutions is specified by the boundary conditions of the problem or the condition that all of the perturbations tend to zero at infinity (the radiation condition).

When all of the kinetic coefficients vanish, the dispersion equation becomes a second-order equation and, consequently, two of its roots are regular with respect to the kinetic coefficients, that is, they are representable in the form of series in nonnegative, not necessarily integral, powers of the kinetic coefficients. The solutions corresponding to them uniformly pass into the solutions of the dispersion equation for an ideal fluid and describe the propagation of perturbation waves.

Three types of motions, the properties of which are substantially different from wave motions, correspond to the six other solutions, which are singular with respect to the viscosity and the diffusion coefficient of the salt. These solutions are expanded in series in integral and fractional negative powers of the kinetic coefficients. Regular functions describe large components of the flow that are waves and vortices. Singular perturbed solutions have not been studied properly before.
Analysis of complete set of regular and singular disturbed solutions is given in [3]. Here we are limited only by acoustic-gravity waves, which are essential for compressible flows.

When the effects of rotation are neglected ($\Omega = 0$), it follows that propagating gravity waves exist in two frequency ranges $\omega \leq N_c$ and $\omega \geq N$. At lower frequencies ($\omega \leq N_c$), they display the properties of internal gravitational waves and, at high frequencies ($\omega \geq N$), approach, as regards their character, acoustic waves in an isotropic medium. In the latter case, two different gravitational-acoustic boundary layers with the characteristic scales

$$
\delta_{b-} = \delta_N \sqrt{2/\sin \theta_\omega}
$$

$$
\delta_{b+} = \delta_N \sqrt{2\sin \theta_\omega \over |1 - g \Lambda/c^2| \sin^2 \theta - \sin^2 \theta_\omega|} \approx \delta_N \sqrt{2\sin \theta_\omega \over \sin^2 \theta - \sin^2 \theta_\omega}
$$

$$
\delta_N = \sqrt{v/N}, \quad \theta_\omega = \arcsin(\omega/N)
$$

are formed on the rigid boundaries in the medium. The first of them is similar to the periodic Stokes flow of a homogeneous fluid, and the second, with parameters, which depend both on the stratification ($N$) as well as on the compressibility ($c$), is spherical in the case of stratified media. Direct calculations of the divergence of the velocity show that the fluid in the boundary-layer flow behaves as an incompressible fluid in spite of the fact that the existence of such a periodic flow is partially due to the effects of compressibility.

The dispersion equation for periodic perturbations of a homogeneous incompressible fluid ($N = \Omega = 0$) has the simplest form $k^2 D_v(k) = 0$. The first factor characterizes pseudowaves $k^2 = 0$ in which all three components of the wave vector $k$ are non-zero. Such motions possess the property $v = p k^2 / \rho$ and taking account of incompressibility leads to $k$ and $v$ having complex values. When spatial motions about a rigid oscillating boundary are considered, a doubly degenerate viscous wave boundary layer arises with a thickness $\delta_b = 2\sqrt{v/\omega}$, consisting of two periodic Stokes waves. The set of governing equations, which does not contain specific equation of state, becomes underdetermined and the particular problem of flow on the oscillating surface become degenerated.

General solutions are illustrated by particular problem solution for flows induced by uniformly moving strip along a rigid plane in a continuously stratified fluid. Another example of flows generated by a slowly oscillating piston in the horizontal plane surface was analyzed numerically and visualized experimentally [4, 5]. Existence of singular disturbed solutions as high gradient extended domains in a fluid interior and their transformation in isolated vortices in domain of their convergence was demonstrated.

### 2. Infinitesimal solutions for a stratified flow around a strip.

Flows induced by a strip of length $L_x$ moving horizontally with velocity $U$ one is most important case in the analysis as exact solutions of 2D Navier–Stokes equations in approximation of an incompressible fluid can be compared with experiments. In this case both the governing equations and boundary conditions are linearized and diffusion effects were neglected. Singular perturbed solutions of the problem reflect before unknown effects of viscosity.

The solution of the equations (6) for stream function is [6]

$$
\Psi(x, z, t) = iU \pi \int_{-\infty}^{\infty} k \sin k a e^{ik(x-Ut)} d\Psi_{kj}(kU, k) dk
$$

where $v_x = \partial \Psi / \partial z$, $v_z = -\partial \Psi / \partial x$
and simplified dispersion equation (4)

\[
\omega^2 \left( k^2 + k_z^2 \right) - N^2 k^2 + i\omega \left( k^2 + k_z^2 \right)^2 = 0
\]

is solved explicitly

\[
k_w^2(\omega, k) = -k^2 + \frac{i\omega}{2\nu} \left[ 1 - \sqrt{1 + \frac{4\nu k^2 N^2}{\omega^3}} \right]
\quad \text{and} \quad
k_t^2(\omega, k) = -k^2 + \frac{i\omega}{2\nu} \left[ 1 + \sqrt{1 + \frac{4\nu k^2 N^2}{\omega^3}} \right]
\]

Regular perturbed roots \( k_w \) describe large scale component of motion, their wave length is proportional to the strip velocity \( U \) and buoyancy period \( T_b \); \( \lambda = U \cdot T_b \). Waves exist in the limit of ideal stratified fluid. Singular disturbed roots characterize small scale component of the fluid motion.

To construct the flow pattern, we used the Simpson method to calculate the values of integrals (7) and the components of the velocity vector at grid nodes; the grid step was chosen from the condition of resolving of the velocity boundary layer (up to 10 points at the transverse scale of singular disturbed solution (9) – \( \delta_u = \nu / U \)). With the help of a code written in the MS Visual C++ shell, the data obtained were used to construct the colored image of various physical fields.

Calculated large scale flow pattern around the strip moving from right to left along the solid plane is presented in Fig. 1. The wave field is formed by matched transient upstream waves ahead the strip, interferometric pattern of superposed stationary lee waves past leading edge of the strip and transient waves ahead of rear edge of the strip and finally downstream stationary attached waves. Sloping rays present crest and troughs of transient waves. Phase surfaces of lee waves are parts of circumferences with centers at the edges of the strip.

![Fig. 1. Pattern of attached internal waves near moving horizontal strip. Bright red points indicate edges of the strip (\( T_b = 6.28 \) s, \( L_x = 40 \) cm, \( U = 0.55 \) cm/s, \( \lambda = 3.5 \) cm).](image)

Structure of the field of the absolute values of the horizontal velocity component around the strip, presented in Fig. 2 contains several typical elements: singularities in the vicinity of the leading and trailing edges, boundary layer of the Prandtl type, and phase surfaces of internal waves. A comparison with results of previous calculations of the flow past a strip shows that vertical curves characterize the positions of the crests and troughs of internal waves for this component of flow velocity.

The sloping curve of zero velocities refers to the outer edge of the boundary layer formed by the sum of regular and singular components of the flow above the strip. In contrast to a uniform fluid, where the boundary-layer thickness \( \delta_u \) depends on the method of its determination (criteria of one-percent and half percent velocity deficit at the boundary are used in practice), the curve of the zero values of velocity along the plate in a stratified fluid rigorously determines the position of the outer edge of the boundary layer. The change in the sign of velocity disturbances is caused by the existence of internal waves in the entire space of the periodic field. The boundary-layer thickness
monotonically increases along the plate. The singularity of the trailing edge is more pronounced than the singularity of the leading edge. The edge singularities are also more clearly expressed in the field of the vertical velocity component (Fig. 2, b). The fluid is pushed away from the plate near the leading edge and returns to the initial level near the trailing edge. The position of the curves of the zero value of the vertical velocity component near the plate is determined by the phase structure of the field of internal waves.

Structure of the field of the absolute values of the horizontal velocity component around the more long strip $L_x = 7.5$ cm, presented in Fig. 2 is similar to previous but the edge singularities became lesser pronounced the other structural components namely the boundary layer of the Prandtl type and internal waves separated from boundary layer on the plane by a zero velocity layer.

A comparison of Figs. 1-3, a and b shows that the flow patterns near the plate in different variables are as different as in the field of internal waves far from the plate. A comparison of the flow patterns shows that effective sources of waves are edge singularities responsible for the shift of density isopleths and emergence of unsteady upstream and steady downstream (lee) internal waves.

![Fig. 2. Modulus of vertical and horizontal components of velocity near a short strip moving from right to left](image)

($T_b = 14$ s, $L_x = 2$ cm, $U = 1$ cm/s, $\lambda = 14$ cm, Fr = 1.12, Re = 200)

The nonlinear terms in the complete set (1) characterize the interaction of both regular and singular perturbed component of the flow. The interaction of singular elements is primarily manifested near the plate, leading to formation of vortex elements and complicating the flow structure. As the velocity shear reaches the maximum values near the plate, the results of interaction of singular components start to manifest themselves already at comparatively low velocities and lead to formation of new structural (vortex) elements.

![Fig. 3. Calculated patterns of horizontal component of fluid around the long horizontal strip](image)

($T_b = 14$ s, $L_x = 7.5$ cm, $U = 1$ cm/s, $\text{Re} = 7500$, Fr $= 0.3$,): a, b) – near and far fields.

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The dimension and scaling analysis of the set (1) show that the list of intrinsic length scales of the non-linear problem includes the buoyancy scale \( \Lambda = d(\text{ln} \rho_0)/dz \), the size of the obstacle (length \( L_x \) or diameter \( D \)), the length of the attached (lee) internal waves \( \lambda = UT_0 = 2\pi U/N \), the transverse length scales of the velocity \( \delta_u = \nu/U \) and density \( \delta_\rho = \kappa_\nu/U \) boundary layers on the rigid surface and thicknesses of internal boundary layers \( \delta_N = \sqrt{\nu N}, \delta_\rho = \kappa_\nu/U \). In this description the flow pattern is characterized by the set of dimensionless parameters, namely the Reynolds number \( \text{Re} = UD/\nu = D/\delta_u \), Mach number \( M = U/c \), internal Froude number \( \text{Fr} = U/ND = \lambda/2\pi D \), Peclet number \( \text{Pe} = UD/\kappa_\nu = D/\delta_\rho \) and ratio of scales \( C = \Lambda/D \) are the ratios of the basic length scales and velocities of the problem.

These scales form strong inequalities \( \Lambda >> D >> \delta_u >> \delta_N \); \( \Lambda >> \lambda >> \delta_u >> \delta_N \) for laboratory and environmental conditions. Experimental methods must provide visualizing the physical fields since a location of the fine interfaces is a priori unknown. Plurality of length scales reflects complexity of the flow structure. To register the elements of the flow field with the scales \( \delta_u, \delta_N \) one has to use field methods with high sensitivity and resolution. Commercial optical Schlieren instruments satisfy these requirements.

3. Experimental Technique. The experiments were performed in a transparent tank 220×40×60 cm with optical illuminators built into the side walls. By the method of continuous displacement, the tank was filled by a stratified solution of common salt. The homogeneity of stratification and the buoyancy period were monitored by observations of fluid oscillations excited by a density marker (wake past free sinking salt crystal); the measurements were performed by a contact electrical conductivity probe, with the marker being formed in the vicinity of the probe sensor. The error of the direct measurement of the buoyancy period was less than 5%.

Above the tank, there was a carriage, which was towed along the guides with a velocity \( U = 0.1 – 15.00 \text{ cm/s} \). A horizontal rectangular strip was attached to the carriage with the help of vertical knives whose lower part was made of thin transparent plastic. The horizontal positions of the strip and of its trajectory with respect to the water surface in the period of pool filling were carefully monitored. The strips used for experiments were 0.1 cm thick, 36.5 cm wide, and had a length \( L_x = 2.5 \) or 7.5 cm. The range of experimental parameters corresponds to the laminar and transitional (vortex) flow regimes.

The observations were performed by an IAB-458 Schlieren instrument with a white light source. For refraction effects caused by light dispersion in the working medium to be reduced, the illumination slot was oriented vertically. The cutting diaphragms forming the Schlieren images were the Foucault knife or a filament, which were also mounted vertically. The method of the Foucault knife is characterized by high sensitivity, while the method with the filament is more informative, because it allows one to distinguish the wave crests and troughs and to visualize thin high-gradient interfaces on the background of intense internal waves [8].

4. Basic experimental results.

Superposition of images visualized by the method with the filament and the calculated flow patterns formed during the motion of the horizontal flat plate moderate velocities shows that the structures of the phase surfaces of internal waves are in good agreement (Fig. 4, a). Differences in details of calculated displacements of fluid particles and real measurements of electrical conductivity (blue line in Fig. 4, b) illustrate role of non-local effects in the laboratory tank of final sizes.

The experimentally observed pattern shows both the advanced unsteady waves (inclined beams ahead of the body) and the steady attached internal waves (deformed semicircles behind the plate whose edges are marked by the arrows). In the lower part of Fig. 5, a, the deformed vertical lines
show the density markers formed by freely submerging crystals of sugar, which were used to visualize the horizontal component of fluid velocity. The points where the markers change the direction of their motion coincide with wave crests and valleys. For low Froude numbers, the most intense disturbance arises ahead of the obstacle, while the disturbance behind the obstacle is weaker.

Fig. 4. Schlieren image of the flow pattern near the long strip and particle displacements in the wave wake: ($T_b = 7.6$ s, $L_x = 7.5$ cm, $U = 3.97$ cm/s, $Re = 2780$, $Fr = 0.63$).

Directly above the plate, where unsteady and steady waves from different sources overlap, the wave field has a complicated structure, and the non-local character of the source is manifested even in the flow field above the short plate. Distortions of the shape of wave surfaces above the plate become less intense with distance (Fig. 5, a). With increasing velocity, the wave length increases, the contact point between the first wave crest and the plate is shifted toward the trailing edge (Fig. 5, b), and a system of short thin disturbances aligned at an angle to the velocity direction is formed near the plate. The system of these disturbances is called the transverse streaky structure. Some streaks are directed along the streamlines

![Image](image1.png)

Fig. 5 Superimposed calculated and experimentally observed flow patterns near the short plate ($L_x = 2.5$ cm):

a) – absolute value of the vertical velocity component (upper part of the figure, the arrows indicate the plate edges) and the Schlieren image of the flow with density markers (lower part of the figure) ($U = 0.17$ cm/s, $T_b = 14$ s, $Re = 42$);

b) – Schlieren image of the flow with superimposed streamlines ($U = 2.3$ cm/sec, $T_b = 7.55$ s, $Re = 575$, $Fr = 1.1$).

As the strip motion velocity is increased, regular streaky structures are grouped into clusters containing secondary short high-gradient interlayers, which form the outer boundary of vortices in the classical vortex street. Because of the suppressing effect of buoyancy forces, the vertical size of the vortex street ceases to grow rather rapidly. The flow is no longer homogeneous along the beam of the Schlieren instrument, which passes parallel to the plate edges, i.e., the flow becomes unstable in the transverse direction and decomposes into a system of three-dimensional vortices. Gradually
stratification suppresses the vertical component of velocity and vortices are transformed into planar structures.

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The angular position of the strip substantially affects the pattern of internal waves and the fine wake structure. The lift force supplementing the drag force leads to displacement and relative shift of the phase surfaces of attached waves. In experiments illustrated in Fig. 7, a the troughs of the first internal wave is located on the leading edge of the plate in the upper half-space and contacts the lower surface at a distance of 0.9 cm from the leading edge. The first crest of the attached wave is adjacent to the trailing edge of the plate in the upper half-space.

The pattern of the streaky structures above and below the inclined plate also becomes essentially asymmetric. No streaks are observed on the upwind side almost along the entire plate, except for the vicinity of its trailing edge. A system of short streaks extended in the direction of motion is adjacent to the domain of the wake contact with strip. On downwind side, the streaky structures are formed along the entire plate, beginning from the leading edge. With the strip velocity increasing the streaky structure is transformed into sloping set of asymmetric vortices presented in Fig 7, b. The phase surfaces pass through the wake split into individual extended interfaces and through shorter inclined streaky structures. The dark and grey curves (wave troughs and crests, respectively), beginning from the second one, come into contact in the wake region. The symmetrized
phase structure of the wave field indices wavy oscillations of the density wake, which submerges as a whole into the valleys and arises in the crests.

**Conclusion.**

Numerical visualization of the full solution of the linearized system of fundamental set of equations for a stratified fluid reveals regular (internal waves, advanced disturbance, wakes) and singular perturbed components of the flow (edge disturbances and boundary layers) existing at any value of velocity. In the experiment, the flow near the strip and far fields also contains large scale (waves and vortices) and thin extended components (high-gradient interfaces and streaky structures). High-gradient interfaces in the wake are observed for all velocities of strip motion. Formation of the streaky structures is caused by the action of edge singularities. At low velocities of plate motion, linear models adequately describe the pattern of internal waves. The wave phase structures are in both qualitative and quantitative agreement with calculated results. The effect of the lift force caused the sloping of the strip is manifested in changes in the phase structure of internal waves and in the shape, size, orientation, and character of evolution of streaky structures and vortex systems.

At large velocities when compressibility effects become important the singular component manifest itself as shock waves. Their analogues in stratified flows are registered both at small and relatively large velocities. Detailed investigation of singular perturbed components in a wide range of flow parameters is subject of future studies.

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