INFLUENCE OF POROSITY ON NONLINEAR PROCESSES IN HYPersonic 
BOUNDARY LAYER

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Introduction

Studying of laminar-turbulent transition is one of the major problems in gas dynamics. By present time transition mechanisms at hypersonic speeds are studied poorly. Physical processes of the transition at hypersonic speeds qualitatively differ from processes at sub- and supersonic speeds. The basic difference is an appearance of the acoustic disturbances predicted by L. M. Mack [1] (so called Mack modes). In two-dimensional boundary layers, beginning from Mach number about \( M \approx 4.5 \) the transition occurs as a result of the amplification of the second mode.

In high-speed flows, the second mode is associated with disturbances of relatively high frequency corresponding to the ultrasonic band. Malmuth et al. [2] assumed that a passive ultrasonically absorptive coating (UAC) of fine porosity may suppress these fluctuations and, at the same time, may not trip the boundary layer owing to roughness effects, i.e. the passive UAC may stabilize the second and higher modes by a disturbance energy extraction mechanism.

The concept was verified in the California Institute of Technology GALCIT T-5 shock tunnel by testing a 5\( ^\circ \) half-angle sharp cone [3]. Detailed investigations of disturbances development on a porous surface on linear stage of transition have been done in [4]. It was shown that porous surface significantly suppress second mode pulsations.

Bispectral analysis was used in [5] for nonlinear process investigation of the transition on porous surface. Perforated metal sheet with regular porosity has been used as a porous covering. It has been shown, that a subharmonic resonance which is one of the basic nonlinear mechanisms on solid surface was suppressed by a porous covering. At the same time insignificant increase of the nonlinear interactions related with the first mode was observed.

In the present paper an influence of porosity on the nonlinear processes in a hypersonic boundary layer is investigated by means of statistical and bispectral analysis. Unlike the previous work the present investigation analyses nonlinear interaction not only in a layer of the maximum rms, but in all the thickness of the boundary layer.

Experimental facilities

The experiments were carried out in the T-326 hypersonic wind tunnel of the ITAM SD RAS. This facility is a blowdown wind tunnel of the ejector type that exhausts to atmosphere. Free stream Mach number was 5.95\( \pm 0.8\% \), total pressure and total temperature were \( T_0 = 390\pm 0.9\% \) K and \( P_0 = 10^6\pm 0.25\% \) Pa, which correspond to a unit Reynolds number \( Re_1 = 12.2 \times 10^6 \) m\(^{-1}\). All experiments are carried out under adiabatic wall model conditions.

The fluctuations in the boundary layer were measured with a custom-made constant-current hot-wire anemometer that has a bandwidth up to 500 kHz. Single-wire probes made from a tungsten wire 5\( \mu \)m in diameter and 1.3mm long were used.

The output signal of the hot wire is split into dc and ac components and both components are digitized separately by means of two 12-bit analog-to-digital converters (ADCs) with a sampling rate of 5 MHz.

The model was a 0.5-m-long steel cone with a 7-deg halfangle. The model is installed at zero angle of attack. The part of the model was a perforated sheet, which constitutes the porous surface; the diameter of the holes in the sheet is 50\( \mu \)m and the spacing between holes is 100\( \mu \)m; the open
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area (porosity) is 0.2.

**Statistical and bispectral analysis**

For detecting of the nonlinearity effects the statistical and bispectral analysis are used. The statistical analysis gives an integral characteristic of the nonlinear processes whereas bispectral method allows to reveal details of the nonlinear interactions.

The following coefficients were calculated for the statistical analysis:

\[ S = \frac{E\left(\left(x_{\text{mean}} - x\right)^3\right)}{\sigma^3} \]  - skewness,

\[ K = \frac{E\left(\left(x_{\text{mean}} - x\right)^4\right)}{\sigma^4} \]  - kurtosis,

where \( E \{ \} \) – expectation function, \( \sigma \) – dispersion, \( x \) – signal.

Skewness shows a deviation of the distribution from normal distribution, for which \( S = 0 \).

Kurtosis shows widening or narrowing of the distribution in comparison to the normal distribution. For the normal distribution kurtosis equals to 3.

Basic definitions for the bispectral analysis are given below [6].

Second and third order moments are calculated as:

\[ c_2 = E\left\{ x(n)x(n+k) \right\} \]

\[ c_{ij}(k,l) = E\left\{ x(n)x(n+k)x(n+l) \right\} \]

where \( x(n) \) – a signal; superscript «*» denotes complex conjugation.

Fourier transform of the second and third order moments gives power spectrum and bispectrum:

\[ P(f) = \sum_k c_2(k)e^{2\pi ki} = E\left\{ X^*(f)X(f) \right\} \]  - power spectrum,

\[ B(f_1, f_2) = \sum_{k,l} c_{ij}(k,l)e^{2\pi i(f_1k+f_2l)} = E\left\{ X^*(f_1+f_2)x(n)x(n) X(f_1)X(f_2) \right\} \]  - bispectrum,

where \( X(f) \) – Fourier transform of the signal \( x(n) \).

Due to the symmetry of the bispectrum it is enough to know its value in a triangle \((0,0), (f_N,0), (f_{N/2},f_N/2)\), where \( f_N \) – Niquist frequency. Bispectrum is zero for the normal distribution.

Bispectrum amplitude depends on the wave amplitude. To avoid this the bispectrum is ordinary normalized to the power spectrum, that leads to the bicoherence:

\[ \text{bic}^2(f_1, f_2) = \frac{\left| B(f_1, f_2) \right|^2}{P(f_1)P(f_2)P(f_1+f_2)} \]

Bicoherence amplitude shows phase locking of three waves \( f_1, f_2 \) and \( f_1+f_2 \). Bicoherence amplitude can be in a range from 0 (independent waves) to 1 (completely phase locked waves).

**Results and discussion**

Mean and rms voltage fluctuations on the hot-wire gauge are shown in Fig. 2 and 3 for a solid and porous surfaces in three various positions. Co-ordinate \( y \) is normalized on a thickness of the boundary layer \( \delta \). In a Fig. 3 a mean voltage normalized on a value of mean voltage at boundary layer edge \( E_0 \). It is seen that in the last positions \( x = 315 \text{ mm} \) (Fig. 2) and \( x = 322 \text{ mm} \) (Fig. 3) profiles of mean and rms voltage essentially differ from each other. The profile of the mean voltage on a solid surface is closer to a turbulent profile, in comparison with a case of porous wall. Fig. 2 shows that the layer of the maximum rms voltage pulsation on a solid surface in last section is shifted more close to a cone wall. But on a porous surface it remains at the same level. However, as well as in case of a solid surface the distribution of pulsations becomes wider in \( y \) direction. That means the transition to turbulence has started on both surfaces. But behavior of mean and pulsation
voltage profiles points out the transition on solid wall is already at more developed stage in comparison with a porosity case.

Fig. 2. RMS voltage fluctuations across boundary layer. (a) – porous surface; (b) – solid surface.

Fig. 3. Mean voltage distribution. (a) – porous surface; (b) – solid surface.

Fig. 4 shows the distribution of skewness and kurtosis across the boundary layer (x = 209 mm for solid surface and x = 181 mm for porous surface). On the porous surface the maximum deviation of these factors exceeds values on the solid surface. Distribution of statistical coefficients in case of porosity is less smooth. Probably, the reason is high level of electric noise presented in a signal during measurements on the porous surface.

Fig. 4. Mean parameters distributions: <e> – rms voltage fluctuation, E – mean voltage, K – kurtosis and S – skewness. (a) – porous surface x = 181 mm; (b) – solid surface x = 209 mm.

Bicoherence for the same sections is shown in Fig. 5. On top and on right of the bicoherence are Fourier spectra. Bicoherence shows waves which are phase locked. If there is a peak at frequencies $f_1$ and $f_2$, that means three waves with frequencies $f_1$, $f_2$ and $f_1+f_2$ are phase locked.

Value $y/\delta = 0.8$ corresponds to the maximum rms voltage fluctuations; value $y/\delta = 0.89$ corresponds to the height where deviation of the kurtosis and skewness is maximum on the porous side (Fig. 5). On porous surface nonlinearity is absent. But on the solid side light nonlinear interactions in low frequency range appears close to the boundary layer edge. Thus nonlinear
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processes on the solid surface has already appeared, in contrast to the porous surface case. The deviation of statistical coefficients shows presence of nonlinearity at the edge of the boundary layer on porous wall (Fig. 4a).

Fig. 5. Bicoherence. (a), (c), (e) – porous surface \( x = 181 \) mm; (b), (d), (f) – solid surface \( x = 209 \) mm.
However bicoherence shows no nonlinear processes. The most likely reason is that statistical coefficients are the integral characteristic and weak nonlinear interactions are summed in kurtosis and skewness calculations. Bicoherence displays concrete nonlinear interactions which can be separately small and, therefore, are not visible in spectra.

Fig. 6 shows the distribution of skewness and kurtosis across the boundary layer ($x = 245$ mm for solid surface and $x = 251$ mm for porous surface).

Deviation of the skewness and kurtosis is increased for both surfaces. Rms fluctuations are increased as well.

![Fig. 6. Mean parameters distributions: $\langle e \rangle$ – rms voltage fluctuation, $E$ – mean voltage, $K$ – kurtosis and $S$ – skewness. (a) – porous surface $x = 251$ mm; (b) – solid surface $x = 245$ mm.]

At the height $y/\delta = 0.5$ nonlinear interactions are observed in low frequency band on the porous wall (Fig. 7a). On solid wall at the same height nonlinear interaction are situated along a line $f_1 + f_2 = f_{II}$, where $f_{II}$ - second mode frequency (at this height Fourier spectrum shows $f_{II} = 290$ kHz, Fig. 7b). Interactions on this line correspond to the subharmonic resonance with detuning [7].

At maximum rms fluctuation height ($y/\delta = 0.8$) nonlinear interactions along the line of subharmonic resonance are increased on solid wall (Fig. 7d). Nonlinearity appears in low frequency band. On porous wall nonlinearity along the line $f_1 + f_2 = f_{II}$ is weak and bicoherence amplitude are smaller then for the solid surface (Fig. 7c). This result has already been observed in [5] and it points out the second mode is suppressed by the porosity. Fourier spectrum on porous surface shows significant amplification of the first mode amplitude (peak at $f \approx 125$ kHz, Fig. 7c). Probably this amplification results in high intensity of nonlinear interactions in low frequency range at the edge of the boundary layer ($y/\delta = 0.95$, Fig. 7e). Bicoherence amplitude on the solid surface at the edge of the boundary layer becomes lower ($y/\delta = 0.95$, Fig. 7f). Thus, if the bicoherence amplitude is maximum in a maximum rms fluctuation layer on solid surface, in case of porosity nonlinear processes are the least intensive in this layer.
Fig. 7. Bicoherence. (a), (c), (e) – porous surface $x = 251$ mm; (b), (d), (f) – solid surface $x = 245$ mm.

Fig. 8 shows the distribution of skewness and kurtosis across the boundary layer ($x = 315$ mm for solid surface and $x = 322$ mm for porous surface). The main feature of the statistical coefficient distribution at these sections is that the distortion of the distribution goes out of the boundary layer. The coefficients become equal to normal distribution values at height $y/\delta \approx 1.6$ mm for the solid surface and $y/\delta \approx 1.4$ mm for the porous surface (Fig. 8). That is distortion on porous side
disappears earlier. Taking into account that rms fluctuation and mean voltage profiles at this section on the solid side are closer to turbulent distributions in comparison to the porous wall (Fig. 2, 3), it can be concluded that evolution of transition is delayed on porous surface.

Bicoherence for these sections is shown in Fig. 9. Nonlinear interactions are presented at any height of the boundary layer on both surfaces. Low frequency interactions are observed at low height on porous surface (Fig. 9a). On the solid wall the interactions are observed almost in all frequency range limited by a line $f_1+f_2=f_{II}$ (Fig. 9b). Close to the maximum rms fluctuation layer bicoherence amplitude is growing on solid side (Fig. 9d). Nonlinear interactions on porous side are mainly concentrated along the line $f_1+f_2=f_{II}$ (Fig. 9c) just as it was in previous section on solid surface (Fig. 7d), but the amplitude here is lower. At the edge of the boundary layer the nonlinear processes are the same on both sides, but the intensity is higher on solid side (Fig. 9e, f). Again nonlinear interactions are limited by the line $f_1+f_2=f_{II}$. Out of the boundary layer at height $y/\delta=1.4$ nonlinear processes on porous surface disappear in contrast to the solid surface case (Fig. 9g, h), that is in a good agreement with statistical coefficient distribution. Going of nonlinear processes out of the boundary layer apparently means that turbulent boundary layer starts to form.

As was mentioned above nonlinear processes evolution is delayed on porous surface. At the same time qualitatively the nonlinear processes are the same on both surfaces. I.e. the damping of the second mode (and as a sequence the damping of subharmonic resonance) delays transition process, but not change basic mechanisms of the nonlinearity. Thus we can suppose that subharmonic resonance plays catalytic role in energy transfer from the mean flow to the low frequency disturbances, just as it observed at subsonic speeds.
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Рис. 9. Bicoherence. (a), (c), (e) – porous surface \( x = 322 \) mm; (b), (d), (f) – solid surface \( x = 315 \) mm.
Conclusions

Data of the nonlinear interactions in hypersonic boundary layer of sharp cone are presented. Difference of the nonlinear processes on the porous and solid surfaces is shown by means of bispectral analysis.

As was shown in previous experiments subharmonic resonance is damped on a porous wall. Nonlinear interactions are more intense out of maximum rms fluctuation layer (lower and over of the layer). Nonlinear processes are delayed on porous surface, but porosity does not change basic mechanisms of the nonlinearity. Such behavior can mean that suharmonic resonance plays catalytic role in energy transfer from the mean flow to the low frequency disturbances, as it happened in subsonic case.

REFERENCES

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