THE CHOICE OF TURBULENCE MODEL DURING ANALYSIS OF METROLOGICAL CHARACTERISTICS OF FLOWMETERS WITH STANDARD ORIFICE


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Variable pressure-drop flowmeters (VPDF), in particular, flowmeters with standard orifice are the basic type of flowmeters used for long-distance pipelines, which makes them very commercially important. Major types of VPDF are standardized, however constant toughening of requirement to accuracy of flow measurement, which was based up to the present exclusively on experimental data, necessitates periodic revision of standards. The experimental study of metrological characteristics of VPDF requires financial expenses and is often associated with significant difficulties of technical character. That is why introduction of numerical methods for analysis of VPDF is the highly actual issue.

The modern level of computational fluid dynamics (CFD) allows solving many practical tasks with sufficient accuracy. Results of numerical study of flow in VPDF are presented in a number of publications [1 – 2]. However there is still no clear idea as to whether it is possible to calculate metrological characteristics of gas and fluid flowmeters using numerical methods, so, as a rule, application of CFD for analysis of flowmeters is of auxiliary character.

On the other hand, there is a number of applications in flow measurement in which employment of numerical method for analysis of VPDF characteristics could be justified. Such applications include using flowmeters under non-standard conditions which quite often arise during assembly of measuring units, expanding of standard scope by both Reynolds number and types of flowmeters, study of flows in flow straighteners, optimal designing of flowmeters.

The peculiarity of numerical analysis of flowmeters’ characteristics is the requirement for high accuracy, which is determined by the error of the experiment itself, and which is specified by GOST and for standard orifices this should not exceed 0,5% [3].

The quality of solution received with use of numerical methods is determined, first of all, by the quality of grid and the chosen model of turbulence. In calculation of VPDF with standard orifice this means the dependence of discharge coefficient, $C$, on grid parameters and chosen model of turbulence, which, in their turn, will be dependant on Reynolds number and the ratio of orifice diameter to the diameter of measuring pipeline (MP), $\beta$. So, in order to evaluate the possibility of CFD methods applicability for calculation of metrological characteristics of VPDF it is necessary to carry out the study of influence of grids and turbulence models on value $C$ in the wide range of $Re$ and $\beta$.

In this paper such a study was carried out in axi-symmetrical and 3D environment for uncompressible stationary flow in the $Re$ range from $5 \cdot 10^5$ to $10^8$ using software system Fluent 6.2. The geometry of MP was chosen according to GOST; the values of relative diameter of orifice were taken as $\beta = 0,56$ and 0,75.

Adequate evaluation of boundary layer plays a major role in calculation the discharge coefficient. In order to render the detailed structure of laminar sublayer, especially when Reynolds numbers are great, high resolution is needed. The quality of grid was evaluated using the parameter $y^+ = y\upsilon^+ / \nu$, where $y$ – the distance from wall to the center of boundary cell, $\upsilon^+ = \sqrt{\tau_w / \rho} - \text{wall shear stress}$, $\nu$, $\rho$ – accordingly, kinematic viscosity and density.

The existing recommendations on grid generation could be unacceptable and values $y^+ \approx 1$
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could be unacceptably high taking into account strict requirements to accuracy of calculation. Several variants of grids differing in sizes of boundary near wall cells and cells in the core were generated in order to get various $y^+$ values, at this ratio between sizes of near wall cells and core cells was preserved. It is well known that the form of cell can substantially influence the solution. That is why grids with different form of cell were also generated: rectangular, oriented along the walls and hybrid, containing both rectangular and triangular cells. Fragments of grids near the orifice hole are presented in Figure 1.

A high concentration of grid, constituting is some variants up to 20 layers, was generated by grid generator Gambit in the boundary layer in the near wall area to increase the resolution. Besides, in the process of solution near wall areas were adapted.

3D analysis was carried out only on "coarse grid" with value $y^+ \leq 8$.

The flow in measuring pipeline with standard orifice was already studied well [4], it is the flow with complex structure, characterized by presence of recirculation zones and jet flow, by large gradients of pressures and velocities. Analysis of turbulent flow with backflow zones requires accurate choice of turbulence model. From the large number of developed up to now models of turbulence it is impossible beforehand, without computational check, to choose the appropriate one, that is why there was carried out a testing of a number of widely used models: two-parameter $k-\varepsilon$ (standard, RNG, realizable), two-parameter $k-\omega$ SST and one-parameter model of Spalart-Allmaras (S-A). Evaluation of applicability of Reynolds stress models (RSM) was carried out.

Results of calculations showed that all used models of turbulence present the same flow structure with pair vortexes before and after the orifice. The difference in models of turbulence is a small difference in the dimension of vortexes and the distance to the reattachment point. All $k-\varepsilon$ models approximately alike render the boundary of jet, outgoing from the front edge of the orifice. By standard $k-\varepsilon$ a somewhat larger section in the jet throat is received. In calculations with SST $k-\omega$ model there is a break near the front edge of orifice and minimal section in the jet throat. Realizable $k-\varepsilon$ and SST $k-\omega$ show a small vortex on the slanted wall of orifice, in case of SST $k-\omega$ the vortex is substantially larger than in case of realizable $k-\varepsilon$. Distribution of static pressure and shear stress in models of $k-\varepsilon$ family practically coincide, and model SST $k-\omega$ predicts higher pressure differential on the orifice than $k-\varepsilon$ models.

Comparison of received stream lines of studied turbulence models near orifice for MP with $\beta = 0.56$ at distances from 0,1 $D$ before orifice and to 0,3 $D$ after orifice at Re = $3,1 \cdot 10^6$ is presented

![Figure 1. Grid near the orifice hole: a) rectangular; b) hybrid.](image-url)
in Figure 2.

Results of preliminary study carried out on coarser grids for $\beta = 0.56$ are presented in Figure 3, in which dependences of discharge coefficient deflection $\delta C = (C/C_{GOST} - 1) \times 100\%$, calculated relative to values of GOST [3] depending on Reynolds number is shown.

The error of $C$ determination for $\beta = 0.56$ when using RNG, realizable and SST $k-\omega$ models on adapted and fine grids doesn’t exceed 0.5% in the range of Reynolds numbers from $5 \cdot 10^5$ to $10^7$. Acceptable results are also received with application of RNG model on «coarse» grid and in 3D environment. Based on the figure the coincidence of 3D results and axisymmetric calculations is satisfactory.

It should be noted that in case of transition from “coarse” grid to “fine” grid the reduction of calculation error is achieved not only due to refinement of grid in boundary layer, but also in the stream core.

The error of $C$ determination when using standard $k-\varepsilon$ model exceeds 1.5%.

So, in the range of Re numbers from $5 \cdot 10^5$ to $1.7 \cdot 10^7$ turbulence models RNG $k-\varepsilon$, realizable $k-\varepsilon$ and SST $k-\omega$ ensure the determination of orifice discharge coefficient with error, not exceeding the limits, specified in [3]. Standard $k-\varepsilon$ model on grids with $y^+ > 1$ is not applicable for calculation of discharge coefficient.

Received results served as the basis for a more detailed study of grid influence on discharge coefficient determination.

Dependences of change of averaged by length value $\bar{y}^+$ on cylindrical surface of orifice hole on cross dimension of boundary cell in laminar sublayer $h_{y1}/D < 10^{-6}$ for standard $k-\varepsilon$ model and $\beta = 0.75$ are presented in Figure 4. The figure shows modeling results over wide spectrum of

Figure 2. Stream lines, received by different turbulence models near orifice at $Re = 3.1 \cdot 10^6$:

a) RNG $k-\varepsilon$; b) realizable $k-\varepsilon$; c) SST $k-\omega$; d) one-parameter S-A.
studied grids and variants of their adaptation by means of Fluent 6.2. Linear character of
dependences for $h_{yl}/D < 10^{-6}$ is distinct practically over the whole range of Re numbers. For
Re = 5.3·10^4 linear change starts from $h_{yl}/D \approx 10^{-5}$. Linear character shows that studied grids
distinguish the laminar sublayer well.

Numerical studies showed nonmonotone character of discharge coefficient $C$ change
depending on the cross step of grid in laminar sublayer for the family of $k$-$\varepsilon$ turbulence models.

Dependences of discharge coefficient $\delta C$ deflection on reciprocal of averaged value $\bar{y}^+$ on the

![Figure 3. Dependence of $\delta C$ on Re value at $\beta = 0.56$: solid lines correspond to results received on adapted grid, for which $y^+ \leq 1$; dotted lines – on "fine grid" ($y^+ \leq 4$); triangular – «coarse» grid ($y^+ \leq 8$); circle – 3D analysis.](image)

![Figure 4. Dependences of $\bar{y}^+$ when $\beta = 0.75$ on cross dimension of near wall cell on the cylindrical surface of the orifice hole, received by standard $k$-$\varepsilon$ model of turbulence:](image)
cylindrical surface of orifice hole under constant Re numbers for the standard k-ε model is presented in Figure 5. The picture illustrates the reduction of δC depending on change of \( \tilde{y}^+ \), characterizing the grid.

For \( \text{Re} = 5.3 \cdot 10^4 \) a minimal value of \( C \) discharge coefficient is observed when \( \tilde{y}^+ \approx 0.025 \), which corresponds to the relative value of cross step of the near wall cell in laminar sublayer \( h_{y/1}/D \approx 7 \cdot 10^{-6} \). The value of the step determines the ratio of approximation and rounding errors in numerical modeling. The absence of minimum on the dependence \( C = f(\tilde{y}^+) \) at higher Re values is conditioned by presence of substantial gradients in laminar sublayer, and, due to it, relative reduction of rounding error.

Satisfactory coincidence of calculated value of discharge coefficient, received on coarse grid with \( \text{Re} > 2 \cdot 10^7 \), with the value, specified by GOST [3] is of random character. It is rather difficult to get the grid independence of solution at the cost of grid size reduction, analogous to presented dependencies when \( \text{Re} = 5.3 \cdot 10^4 \) and \( \text{Re} = 3.2 \cdot 10^5 \) in the Figure 4 for a wide range of Re numbers. Received results show, that there is a wide range of \( \tilde{y}^+ \) when \( \text{Re} > 10^6 \), in which metrological requirements are fulfilled already when \( h_{y/1}/D \approx 4 \cdot 10^{-7} \). Moreover, in the solution with the reduction of grid step \( (h_{y/1}/D < 2 \cdot 10^{-7}) \) insignificant fluctuations in the neighborhood of maximal values of shear stress on the wall of MP after orifice are observed.

Hybrid and rectangular square grids show the same results which is quite logical, as the flow near the orifice is not oriented along limiting walls. Due to it both algorithms of Cell-Based and Node-Based gradients calculation (symbols “○” for \( \text{Re} = 5.3 \cdot 10^4 \) and \( \text{Re} = 3.2 \cdot 10^5 \); symbols “□” for \( \text{Re} = 2.5 \cdot 10^7 \) and \( \text{Re} = 4.8 \cdot 10^7 \) in Figure 4,5) give the same results.

The influence of approximation order of partial derivatives on the accuracy of calculation of metrological characteristics was studied as well. Calculation results, received by means of approximation of 1-st and 2-nd order (2-nd order - symbols “○” in Figure 4,5), practically don’t differ. It should be noted, that application of the second order of approximation on a grid when relative cross step of the 1st cell of laminar sublayer \( (h_{y/1}/D) \) less than \( 4 \cdot 10^{-7} \) at \( \text{Re} > 10^7 \) leads to appearance of insignificant fluctuations in the solution as well.

The conclusion on applicability of turbulence models is drawn from Figure 6, is which dependence of \( \delta C \) on \( \text{Re} \) number for different turbulence model is presented. The range of acceptable,
from metrological point of view, values of $\delta C$ is distinguished in the figure.

The calculation results by RNG $k-\varepsilon$ and realizable $k-\varepsilon$ models presented in this figure, were received on a very fine grid with the purpose of receiving grid independence like it was done for standard $k-\varepsilon$ model. The distinctive feature of standard $k-\varepsilon$ model is that it converges stably under any modes of flowing and on any grid.

In contrast to $k-\varepsilon$ family of models the SST $k - \omega$ model and one-parameter (S-A) model work well on coarser grids. For all modes of flowing the limiting value $h_{y^+}/D$ for these models is the value $6\cdot10^6$, at which the solution converges.

RSM model showed satisfactory results in respect to the calculation of metrological results in the narrow range of Re numbers ($5\cdot10^6 \leq \text{Re} \leq 10^7$). It should be noted that this model shows

$$\text{Re} = 5.3\cdot10^4; \quad 3.2\cdot10^5; \quad 2\cdot10^6; \quad 5.1\cdot10^6; \quad 2.5\cdot10^7; \quad 4.8\cdot10^7.$$
fluctuations in solution. Besides, RSM model is very sensitive to any grid skewness.

RSM model showed satisfactory results in respect to the calculation of metrological results in the narrow range of Re numbers \((5 \cdot 10^6 \leq \text{Re} \leq 10^7)\). It should be noted that this model shows fluctuations in solution. Besides, RSM model is very sensitive to any grid skewness.

The studies allow to conclude that in the wide range of Re numbers standard \(k-\varepsilon\), RNG \(k-\varepsilon\), realizable \(k-\varepsilon\), SST \(k-\omega\) and S-A turbulence models, despite the differences in detailed structure of flow, ensure the determination of discharge coefficient with error, not exceeding the limits, specified by GOST [3]. During generation of grid for calculation of discharge coefficient by \(k-\varepsilon\) family of turbulence models, at least, the condition \(y^+ \leq 0,025\) on the cylindrical surface of the orifice hole should be fulfilled.

REFERENCES

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