NONLINEAR EVOLUTION OF DISTURBANCES IN HYPersonic SHOCK LAYER

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Introduction

Viscous shock layers are formed when a flying vehicle moves with a high velocity in the upper layers of the atmosphere. The viscous shock layer consists of a thick boundary layer and a thin zone of inviscid flow behind the bow shock wave and is characterized by intense viscous-inviscid interaction (interaction parameter $\chi = M^3_{\infty}/\sqrt{\text{Re}_{\infty}} > 4$). Studying the evolution of disturbances and understanding the mechanisms of viscous shock layer instability are necessary conditions for the development of effective methods for predicting the laminar-turbulent transition on flying vehicles in a hypersonic flow.

The analysis of the laminar-turbulent transition is conventionally divided into three stages: 1) problem of receptivity and excitation of instability waves by external disturbances; 2) exponential growth of small perturbations; 3) nonlinear interaction of disturbances and stochastization leading to the transition to turbulence. The first and second stages of the laminar-turbulent transition have been successfully studied with the help of the linear theory of hydrodynamic stability [1-2]. In a hypersonic shock layer, however, the flow is not parallel, there exist considerable divergence of the flow and streamwise pressure gradient, the bow shock wave is located rather close to the boundary layer, and instability waves can be excited not only through the receptivity mechanism but also by direct intensification of disturbances in the shock wave. In this case, the use of direct numerical simulations of all stages of the transition, from receptivity to nonlinear interaction, seems to be preferable.

There are some recent publications [3-7] where the problems of receptivity and evolution of disturbances in supersonic and moderately hypersonic flows were solved by means of direct numerical simulations on the basis of solving the full unsteady Navier-Stokes equations. The acoustic mode of instability was demonstrated to dominate in the boundary layer at Mach numbers $M_{\infty} < 10$. In these investigations, however, the flow parameters corresponded to the boundary layer with the shock wave being sufficiently far from the upper boundary of the viscous flow rather than to a hypersonic shock layer.

The processes of disturbance emergence and evolution in the shock layer are essentially different from those inherent in supersonic near-wall flows with moderate Mach numbers. Recent numerical and experimental research [8-9] revealed a pioneering result that the vortex mode of instability dominates in the shock layer in the flow past a flat plate at $M_{\infty} = 21$ under the action of acoustic disturbances of the external flow. Simulations of external acoustic waves with low initial amplitudes turned out to be in good agreement with calculations by the locally parallel stability theory with allowance for the shock-wave effect [10], with the usual asymptotic boundary conditions replaced by conditions on the shock wave.

The present paper describes direct numerical simulations of nonlinear evolution of perturbations in the shock layer subjected to slow-mode acoustic disturbances of the external flow. Distortion of the mean flow by perturbations and nonlinear self-induced action of spectral modes were studied by varying the initial amplitude of external acoustic waves.
Section IV

Formulation of the problem

In the present work, we modeled the evolution of two-dimensional perturbations, which are the most unstable ones at high Mach numbers. The numerical study was performed by means of direct numerical simulations with the use of in-house software based on solving the full two-dimensional unsteady Navier-Stokes equations with high-order shock-capturing schemes [11].

The computational domain was a rectangle, some part of its lower side coinciding with the plate surface. The left (inflow) boundary is located at a distance of a few computational cells upstream from its leading edge. The height of the computational domain is chosen such that the bow shock emanating from the leading edge does not interact with the upper boundary. The right (outflow) boundary is moved downstream from the trailing edge so that the flow in the exit cross section is fully supersonic.

A steady flow was first calculated with the boundary conditions on the left and upper boundaries being set in the form of a uniform hypersonic flow directed along the \( x \) axis. On the right boundary, the solution was extrapolated from inside the computational domain. As the rarefaction effect was rather considerable in the problem considered the boundary conditions on the plate surface took into account the velocity slip and temperature jump [12]. The results of simulations with \( M_\infty = 21 \), \( \text{Re}_L = 1.44 \times 10^5 \) are in good agreement with density in the shock layer measured in experiments by the method of electron-beam fluorescence [8].

Then the problem of interaction of a viscous shock layer with external acoustic disturbances propagating in the streamwise direction was solved. The variables on the left boundary were set as a superposition of the steady main flow and a planar monochromatic acoustic wave:

\[
(u', v', p', \rho') = (u', v', p', \rho') \exp[i(k_x x + k_y y - \omega t)],
\]

where \( [u'], [v'], [p'], [\rho'] \) are the dimensionless amplitudes of velocity, pressure, and density fluctuations \( \delta u = \delta \rho = A, \delta v = -A \cos \theta, \delta p = A \sin \theta \), written for a slow acoustic wave, \( \theta \) is the angle of wave propagation, which is counted in the clockwise direction from the \( x \) axis, \( t \) is the time, \( k_x = k \cos \theta, k_y = -k \sin \theta \) are the components of the wave vector related to the dimensionless frequency \( \omega = 2\pi f L / c_\infty \) by the dispersion expression \( k = \omega / (M_\infty \cos \theta \pm 1) \), and \( c_\infty \) is the free-stream velocity of sound. In the dimensionless relations, density fluctuations are normalized to \( \rho_\infty \), velocity fluctuations are normalized to the velocity of sound \( c_\infty \), pressure fluctuations are normalized to \( \rho_\infty c_\infty^2 \), and geometric dimensions are normalized to the plate length \( L \). Zero temperature fluctuations on the surface were assumed. When disturbances were inserted into the flow, the Navier-Stokes equations were integrated until the unsteady solution reached a steady periodic regime.

Results

The simulations were performed for amplitudes of slow external acoustic disturbances \( A = 0.003 \div 0.6 \) (\( A = 0.04 \) corresponds to the maximum values of the relative density fluctuations summarized over the spectrum, which were measured in the free stream in the wind-tunnel experiment [8]). All subsequent computations were performed with the following flow parameters: \( M_\infty = 21 \), Reynolds number per meter \( \text{Re}_{1\infty} = 6 \cdot 10^5 \, \text{m}^{-1} \), stagnation temperature \( T_0 = 1200 \, \text{K} \), temperature of the model surface \( T_w = 300 \, \text{K} \), angle of propagation of the slow external acoustic wave \( \theta = 0^\circ \), and frequency of this wave \( f = 38.4 \, \text{kHz} \).
Figure 1 shows the isolines of the total density gradient (mean density $\langle \rho \rangle$ plus density fluctuations $\rho'$) or the computed Schlieren visualization for disturbances with low $A = 0.03$ (Fig. 1a) and high $A = 0.6$ (Fig. 1b) initial amplitudes. The solution of the unsteady problem for the flow with low initial amplitude $A$ is seen to be only slightly different from the solution of the steady problem, and the cross-sectional profiles of the total density (Fig. 1c) are similar to the density profiles of the steady flow [8]. For flows with high initial amplitude $A$, the computed Schlieren visualization reveals strong deformation of the bow shock wave and the emergence of inhomogeneities near the boundary-layer edge and in the free stream. Therefore, the cross-sectional profiles of the total density (Fig. 1d) differ substantially from the density profiles in the steady flow.

![Fig. 1. Isolines of the density gradient (mean flow plus disturbances) for low initial amplitude $A = 0.03$ (a), for high initial amplitude $A = 0.6$ (b), and the corresponding density profiles in the cross sections $x = 0.5$ (c) and $x = 0.9$ (d).](image)

The temperature fields including the mean field and disturbances are plotted in Fig. 2. High-amplitude external acoustic waves give rise to ridges in the vicinity of the boundary-layer edge (Fig. 2b), which move faster than the structures inside the boundary layer. As a result, slow roll-up of the ridges is observed. High-amplitude disturbances induce local separation regions near the plate surface, which are absent in the flow disturbed by a low-amplitude acoustic wave (Fig. 2a). The streamlines in Fig. 3 demonstrate the presence and the shape of local separation regions for the case of high-amplitude disturbances and the absence of these zones for the linear case.

![Fig. 2. Temperature fields (mean flow plus disturbances) for $A = 0.03$ (a) and $A = 0.6$ (b).](image)
Figure 4 illustrates the changes in the mean density profile $\langle \rho \rangle$ induced by external acoustic waves with different amplitudes. For flows with low initial amplitudes $A < 0.1$, the mean density profiles in cross sections $x=\text{const}$ coincide with the density profiles in the corresponding cross sections of the steady problem, i.e., the mean flow is not distorted. For flows with initial amplitudes $A > 0.1$, the cross-sectional profiles of mean density spread in the normal direction, up to formation of the second shock on the boundary-layer edge.

High-amplitude initial disturbances also induce noticeable changes in the fluctuating parameters of the flow in the shock layer. Figure 5 shows the root-mean-square profiles of density fluctuations in the cross section $x=0.5$. For flows with low initial amplitudes $A < 0.1$, the profiles have a typical shape with two maximums; the greater maximum is located on the shock wave, and the smaller maximum coincides with the boundary-layer edge. For flows with $A > 0.1$, the maximums of fluctuations merge together and form one maximum with an elevated amplitude. Thus, disturbances occupy the entire area between the shock wave and the boundary-layer edge.
Figure 5. Root-mean-square profiles of density fluctuations in the cross section $x=0.5$ for different initial amplitudes $A$.

Figure 6 shows the amplitude of density fluctuations on the boundary-layer edge in the cross section $x=0.5$ as a function of the initial amplitude of the slow external acoustic wave. At low initial amplitudes ($A < 0.1$), the linear growth of the function $\rho'(A)$ is retarded as the amplitude of the initial disturbance is increased and finally tends to a constant value ($A > 0.2$), i.e., nonlinear saturation of the amplitude at the fundamental frequency is observed. Thus, the amplitude of disturbances generated in the shock layer was found to be a linear function of $A$ within the range indicated above.

For a low amplitude of the slow external acoustic wave ($A = 0.03$), the result of direct numerical simulations were compared with the linear theory of stability with allowance for the shock wave
[10] and with the results of the wind-tunnel experiment [8] (Fig. 7). It is seen that the results of direct numerical simulations for the slow mode are in good agreement with experimental measurements and wi

Fig. 7. Growth rates of density fluctuations on the shock wave: --- - linear theory of stability with allowance for the shock-wave effect [10]; ∆ - experiment [8]; ⎯ - DNS for slow acoustic wave and ⎯ - DNS for fast acoustic wave.

Linear and nonlinear propagation of disturbances in the shock layer is illustrated in Fig. 8, which shows the time evolution of density fluctuations in the cross section $x=0.5$ for different amplitudes of the initial perturbation. For flows with initial amplitudes $A < 0.1$, the signal is rigorously harmonic both in the free stream and in the shock layer (Fig. 8a and b). For flows with initial amplitudes $A > 0.1$, peaks of fluctuations are observed already in the free stream; in the shock layer, these peaks become more abrupt (Fig. 8c and d).
The spectral analysis of density fluctuations is illustrated in Fig. 9. These spectra are normalized to the amplitude of the fundamental harmonic corresponding to a frequency of 38.4 kHz. Nonlinear effects are responsible for the growth of the second harmonic, as the analysis revealed. For $A=0.6$, the amplitude of the second harmonic reaches one third of the amplitude of the fundamental harmonic.
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The rapid growth of the amplitude of the second harmonic is plotted in Fig. 10 as a function of the amplitude of the initial disturbance.

Conclusion

The possibility of computing nonlinear effects is demonstrated on the basis of direct numerical simulations of the evolution of disturbances in a hypersonic viscous shock layer on a flat plate under the action of slow external acoustic waves of different amplitudes.

For low amplitudes of external acoustic waves, the DNS results are in good agreement with the predictions of the linear theory of stability with allowance for the shock wave.

Computations for high amplitudes of initial disturbances display typical features of nonlinearity: mean flow distortion, formation of local separation regions near the plate surface, nonlinear saturation of the fundamental harmonic amplitude, and rapid growth of the second harmonic amplitude.

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