LAMINAR–TURBULENT TRANSITION IN THE BOUNDARY LAYER ON CONES IN A HYPersonic FLOW AT HIGH REYNOLDS NUMBERS PER METER


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Introduction.

The separation properties of a hypersonic boundary layer passing from the laminar to the turbulent state at junctions of various elements of flying vehicles (inlets, flaps, etc.) are often responsible for the flow pattern around the body as a whole and its aerodynamic characteristics.

Boundary-layer stability and laminar–turbulent transition have been studied in many papers. Because of the restrictions on the paper volume, the full state of the art of this problem cannot be discussed in detail (see [1–7]). Up to now, for free-stream Mach numbers \( M_\infty > 10 \), the range of natural (for promising hypersonic vehicles) free-stream Reynolds numbers could not be completely reproduced in experimental facilities (see, e.g. [8]), and the main source of data on the transition were in-flight results. Unlike from previous works on this subject, the data described below were obtained for stagnation parameters that have not yet been reproduced in a gas-dynamic experiment. Present investigation was aimed at obtaining data on the laminar–turbulent transition of the boundary layer flow on a cone at high values of both Mach numbers and Reynolds numbers per meter.

The study was performed in an A-1 piston-driven two-step facility with adiabatic compression. A high pressure in the settling chamber of the A-1 facility \( (P_0 \leq 1000 \text{ MPa}) \) and a rather low temperature \( (T_0=1200–2000 \text{ K}) \) make it possible to obtain record Reynolds numbers per meter \( (Re_1=\rho_\infty v_\infty/\mu_\infty) \) at the nozzle exit. Even for comparatively small model sizes, this allows one to obtain, for \( M_\infty \approx 8–18 \), Reynolds numbers close to natural ones (for flight of promising hypersonic vehicles) and to observe (without artificial tripping) the boundary-layer transition from the laminar to the turbulent state at distances of the order of 30–50 mm from the tip of the model tested. The test time (20–200 msec) and the small size of the model allow one to assume the flow on all parts of the examined model to be steady and to receive several points during one run.

The A-1 layout and its operation principle were described in detail in [9]. The facility has the following limiting parameters: pressure in the settling chamber \( P_0 = 1000 \text{ MPa} \), maximum stagnation temperature \( T_0=2000 \text{ K} \), settling chamber volume \( V=50 \text{ cm}^3 \), nozzle-exit diameter \( d_\infty = 8–40 \text{ mm} \), Mach number at the nozzle exit \( M_\infty = 7–20 \), maximum Reynolds number per meter at the nozzle exit \( Re_1 = 240 \times 10^6 \text{ m}^{-1} \), test time with the maximum pressure \( \tau=20–300 \text{ msec} \) (with near rectangular profile), operation gases \( N_2, \text{ air, argon} \) etc.

The facility has a typical gas-dynamic duct for conventional wind tunnels a conical nozzle (with inner surface roughness less than 1 \( \mu \text{m} \)), test section with transparent optical walls, diffuser, and vacuum tank with a volume of 0.2 \( \text{m}^3 \) evacuated to a pressure of approximately 5 kPa. The use of nozzle inserts with throat diameters ranging from 1.0 to 0.3 mm allows obtaining flows with Mach numbers \( M_\infty = 8–18 \).

Organization of Experiments.

The test gas was commercial nitrogen supplied in liquid form in Dewar flasks. After evaporation nitrogen was compressed to 15 MPa and was stored in standard gas holders at this pressure and room temperature, which varied within \( T_00 = 288–294 \text{ K} \).

The experiments are performed with Mach numbers at the nozzle exit \( M_\infty = 12\pm15 \) and
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pressures in the settling chamber \( P_0 = 60 \pm 620 \) MPa. These values of parameters allow obtaining Reynolds numbers per meter near the cone surface equal to \( \text{Re}_1 = (53 \pm 200) \times 10^6 \) m\(^{-1}\). The transition occurs at Reynolds numbers \( \text{Re}_1 = (2.3 \pm 7.4) \times 10^6 \).

The hypersonic flow was obtained in two conical nozzles with nozzle angle of \( 8^\circ \), and exit diameter of 34 mm. The first nozzle had the throat insert made of tungsten and the throat diameter \( d^* = 0.805 \) mm. In overall series experiments we did not notice any changes in the throat diameter of this nozzle. The throat diameter of the second nozzle insert made of rhenium has progressively increased in series of experiments from 0.675 to 0.7 mm.

The study was performed with three models in the form of a steel cone with an apex angle of \( 15^\circ \). All models were set on a cylindrical holder 13 mm in diameter. The roughness of the model surface was less than 1 µm and the rounding diameter of the model tip was 0.3 mm. The model was mounted at the center of the flow, and its tip was inside the nozzle at a distance of 1–5 mm from the nozzle exit.

To reach the transition on small-size models, which could be tested in A-1, the first series of tests was performed with model, which had a rectangular ring-shaped step 1.4 mm high and 4 mm wide located at the cone base at the point of junction with the holder. The length of the cone generatrix was \( L_0 = 48.2 \) mm. As the transition on this model was registered at a distance of 30–40 mm from the model tip, the ring-shaped step was replaced in the second series of tests by a step 0.8 mm high, the length of the cone generatrix was \( L_0 = 48.5 \) mm. The third (largest) series of tests was performed with model \( (L_0 = 49.3 \) mm) where the conical part joined the holder without any step.

The Schlieren pictures (shadowgraphs) of interaction of the hypersonic flow with the model were taken by a “Karl Zeiss” device for hairline photographing, in combination with a sweep camera and stroboscopic lighting. The lighting generator ensured a prescribed number of frames with a filming frequency of 75–90 Hz, the time of exposure of one frame being \( 0.5 \times 10^{-6} \) sec. The angular velocity of the sweep camera was chosen to be as low as possible without overlapping of frames.

The static parameters of the gas in low-pressure and high-pressure receivers were measured and the pressure transducers were calibrated by standard test manometers with a class of accuracy of 0.4. The test-gas pressure in the settling chamber and its variations were recorded by a manganin gauge specially developed for diagnosing the test gas compressed in the settling chamber [10] and included into the active arm of a conventional bridging scheme. The pressure in facility elements was registered by quartz gauges with a specially developed structure, which were equipped by wide-band charge amplifier with a low-resistance output. The signals from the gauges were recorded by an N-115 loop oscillograph.

Figure 1 shows the pressure oscillograms. (In this experiment, the maximum value of \( P_0(t) \) was 395 MPa.) The numbers 1–9 indicate the time instants: the signal of the photodiode recording the stroboscope flashes, which correspond to the numbers of the frames. The first increase of pressure in the settling chamber corresponds to adiabatic compression of the test gas by the heavy piston of the first stage of the facility. After automatic initiation of the second stage, the test gas is additionally compressed by the piston of the pressure multiplier to a necessary maximum pressure and is displaced from the settling chamber. In these experiments the throat diameter was
sufficiently large for such a settling chamber. For this reason, the pressure multiplier “does not have enough time” to follow gas exhaustion (between frames 6 and 7), even so the pressure during the time interval between frame Nos. 5 and 7 was sustained constant within ±5%. Therefore, the exhaustion process may be considered as quasi-steady at each time instant of the filming and corresponding to the pressure measured in the settling chamber. The pressure oscillogram $P_m$ refers to the pressure on the big piston of the second stage multiplier. When the multiplier stock meets the settling chamber wall, a jump of the pressure $P_m$ is recorded, which coincides in time with a drastic decrease in the settling chamber pressure $P_0$.

Nozzle startup occurs already at the stage of gas compression by the first stage as soon as the critical pressure is reached in the settling chamber and the gas escapes to the test section of the gas-dynamic duct of the facility.

The measurements performed in [11] showed that the heat losses toward the walls may be neglected for the goals of the present work. Therefore the current parameters of the flow at the nozzle exit (static temperature $T_\infty$, pressure $P_\infty$, and Reynolds number $Re_1\infty$) for all time instants (frames) were calculated on the basis of the measured initial parameters $P_{00}$, $T_{00}$, and $P_0(t)$ with the model of adiabatic compression and expansion of the gas. For the same reason and by virtue of the short duration of the test time, the model-surface temperature $T_w$ may be assumed to be equal to room temperature $T_{00}$, and its effect on the boundary layer may be neglected. The calculations were performed by formulas derived in [12] and a special code developed by the authors of the paper.

**Results of Studying the Boundary-Layer Transition.**

Figure 2 shows the typical frames of the Schlieren pictures of flow around the model. The

![Fig. 2. Flow pattern around the cone: $P_0 = 348.5$ MPa and a step 1.4 mm high (a); $P_0 = 201.5$ MPa and a step 0.8 mm high (b); $P_0 = 459.0$ MPa and no step (c); $P_0 = 589.7$ MPa and no step (d); the arrows indicate the beginning of the transition zone.](image-url)
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Foucault knife of the optical system was mounted horizontally. Zones of a purely laminar flow, laminar–turbulent transition, and accelerated increase in thickness of the turbulent boundary layer can be distinguished. An analysis of the flow patterns suggests that an increase in pressure in the settling chamber shifts the beginning of the laminar–turbulent transition zone (marked by arrows) toward the model tip (cf. consecutive frames of one test (Figs. 2c and 2d)). The beginning of the transition zone was determined as the point with a jump (inflection) of density caused by boundary-layer turbulization [13].

For frames in Fig. 2, the calculated pressure at the nozzle exit was $P_\infty = 7–27$ mm Hg, whereas the pressure in the test section at the beginning of exhaustion was $P_\infty = 20–40$ mm Hg. Oblique waves arising at the nozzle edge owing to jet overexpansion obviously cross the boundary layer downstream of the transition beginning. The pressure generated by these shock waves calculated by the angle of their inclination is near equal to or lower than the pressure behind the bow shock wave; hence these shock waves did not affect the transition in our experiments.

At high pressures in the settling chamber, owing to real gas effects and to the fact that the test gas in the A-1 facility is obtained by adiabatic compression, the stagnation temperature, the flow Mach numbers at the nozzle exit $M_\infty$ and above the model surface $M_e$, and the Reynolds number per meter $Re_1e$ depend on the pressure in the settling chamber. Figure 3 shows the Mach numbers and the Reynolds number per meter $Re_1e$ at the nozzle exit as functions of pressure for two nozzles of various throat diameters. A small scatter in the values of $M_\infty$ and $Re_1e$ can be attributed to variations of the initial (room) temperature of the test gas and to the error in measuring the initial and current pressures; for $M_e$, another reason is the error in measuring the angles of inclination of the shock wave to the flow. It follows from Fig. 3, that the values reached in the experiments were $M_\infty = 12±15$, as was calculated by the above-mentioned code with allowance for real gas effects and boundary-layer displacement thickness.

Above the model surface, the Mach numbers $M_e$ (8.5÷10), Reynolds numbers $Re_1e$, density $\rho_e$, velocity $v_e$, and viscosity of the medium $\mu_e$ were calculated on the basis of the experimentally determined angles of inclination $\varphi$ of the bow shock wave toward the flow and the cone apex angle $\beta$, with the use of formulas for an inviscid flow around the cone derived in [14–16].

By introducing the dimensionless flow velocity $\eta$ as the ratio of velocity to its maximum possible value, the dimensionless temperature as the ratio of temperature to the free-stream value $(\theta = T/T_\infty)$, and the degree of compression is the shock wave $\sigma = \rho/\rho_\infty$, we can express the Reynolds number above the cone surface $Re_1e$ via $Re_1e = Re_1\infty \cdot \eta_e \sigma_e / \theta_e$. Here: $\eta_e = \eta_\infty \cdot \cos \varphi / \cos^2(\varphi - \beta)$. The Mach number above the cone surface was determined by the relation: $M_e = M_\infty \cdot \eta_e / \sqrt{\theta}$. 

Fig. 3. Dependences $M_\infty$, $M_e$, and $Re_1$ on pressure in the settling chamber.
The length $L_{tr}$ in the expression for $Re_{tr} \ (Re_{tr} = Re_{1e} L_{tr})$ was measured along the cone generatrix from the model tip to the beginning of the laminar – turbulent transition zone determined from the photographs with the accuracy about to 3%. The real angle of attack was determined with allowance for the correction.

The pressure of nitrogen in the settling chamber at the time instants of photographing $P_0(t)$ was determined from oscillograms similar to those shown in Fig.1; in the present series of tests, it varied from 63 to 620 MPa with an error smaller than 2%. The temperature $T_0(t)$ was determined by calculations and varied within 935÷1650 K. The Reynolds number per meter at the nozzle exit $Re_{1e}$ was $(40÷120) \cdot 10^6$ m$^{-1}$ and $Re_{1e} = \frac{\rho v}{\mu} = (50÷200) \cdot 10^6$ m$^{-1}$ behind the bow shock wave above the model.

Figures 4 show the Reynolds numbers per meter behind the shock wave above the model surface $Re_{1e}$ and the distances $L_{tr}$ between the model tip and the transition point as functions of the pressure in the settling chamber for two diameters of nozzle throat. It should be noted that the values of $L_{tr}$ for models without the step are higher than those for models with the step. The corresponding experimental values for $d^* = 0.805$ mm are located close to the straight lines plotted by the least squares technique:

\begin{align*}
L_n &= (47.4 - 2.85 \cdot 10^{-2} P_0), \quad (1) \\
L_n &= (42.3 - 2.26 \cdot 10^{-2} P_0), \quad (2)
\end{align*}

i.e., the presence of the step leads to a significant decrease in $L_n$. (Comp. eqs. (1), and (2)). $P_0$ is measured in megapascals, $L_{tr}$ in millimeters.

A comparison of the data obtained with available results on the boundary-layer flow transition \cite{1–4, 6} shows that an increase in pressure in the settling chamber leads to an increased in Reynolds numbers per meter $Re_{1e}$ which, in turn, leads to a monotonic (though slowing down) increase in the Reynolds numbers $Re_{tr}$ corresponding to the beginning of the transition. Thereby, the values $L_{tr}$ decrease, and the transition zone is shifted toward the model tip. This effect was already noted, e.g.,

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in [1, 2], but it was observed at Mach numbers M at level of 2÷4 and in a different range of Reynolds numbers per meter Re_{le} (less than 74·10^6 m⁻¹).

The Figure 5 shows the disposition of our experiments in A-1 relative possibilities of reproducing M_∞ and Re by others facilities.

Figure 5. The disposition of our experiments in A-1 relative possibilities of reproducing M_∞ and Re by others facilities.

It should be noted that the transition Reynolds numbers obtained in this work are lower than those in other papers. A possible reason is the difference in test techniques. Pressure or heat-flux measurements are normally performed. Thus, the results obtained in the present work, apparently, refer to the beginning of the transition.

Conclusions.

The process considered is a sophisticated phenomenon affected by many factors; therefore, the results described are typical of this facility and particular conditions of the flow. Nevertheless, owing to capability of A-1, for the first time, these experiments with appropriate Mach numbers have reproduced Reynolds numbers typical of promising hypersonic flying vehicles. Therefore, the study in this field will be continued, in particular, the range of pressures in the settling chamber will be expanded from 620 to 800–900 MPa for obtaining the values of Re_{le} at a level of 240·10^6 m⁻¹, and similar experiments will be performed with other models at higher Mach numbers.

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