PARAMETRIC OPTIMIZATION OF THE MAGNUS WIND TURBINE
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A promising branch of power production is wind-power engineering, operating similarly to
hydropower industry with renewable energy sources [1]. Utilization of wind turbines (WTs) is fuel-
free and does not need lengthy power transmission lines which are rather expensive. Also, a benefit
of WTs is their operational and ecological safety.

There are several shortcomings of traditional blade turbines which are presently used. The
major of them is their low effectiveness at the most repeatable wind velocities \( V < 6 \) to \( 7 \) m/s
because of small lift coefficient of a blade, \( C_y \leq 1 \). Under such conditions, the power coefficient of
WT drops rapidly from its maximum at \( V \approx 8 \) m/s to zero at about \( V = 4 \) m/s. On the other hand, at
\( V > 25 \) m/s the blade WTs can hardly be operated through the risk of blades breakage. Alternatively, the Magnus WTs which are equipped with rotating cylinders instead of blades can be
used for wind-power production [2], see Fig. 1. Some recent research data on their characteristics
are reported in what follows.

Main parameters of the Magnus wind turbine. Experimental studies on the Magnus WT
were performed in T-324 subsonic wind tunnel of ITAM SB RAS. The windwheel diameter \( D \) of
the test model was in the range from 1.3 to 2.0 meters, the number of rotating cylinders \( i \) varied
from 3 to 6, the cylinders diameter \( d \) was equal to 0.05 and 0.06 meters, their aspect ratio \( \lambda \) changed
from 5 to 14 and speed of rotation was up to 8000 rpm. The cylinders were equipped with end
plates, their relative diameter \( C = d_v/d = 2 \) being close to its optimum value. The WT model was
examined in 3.6 to 3.6 meters cross section of the wind tunnel at the oncoming-flow velocity \( V \)
from 1.5 to 3.5 m/s, in certain cases \( V \) was increased up to 4.5 m/s. Research results obtained at
\( \lambda \leq 10.7 \) were given earlier in [3], in the present case we concentrate most of all on the Magnus WT
characteristics at \( \lambda = 14 \).

The windwheel power was determined through approximation of experimental data as
\[
N_w = M_w \omega_w = 0.55 \cdot 10^{-3} G^2 n_w^2,
\]  

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where \( \omega_w \) (1/s) and \( n_w \) (rpm) stand for frequency and speed of the windwheel rotation, \( G \) is the calibration coefficient and \( M_w \) is the windwheel torque, that is

\[
M_w = F^* b . \tag{2}
\]

In the last expression, \( F^* \) is the resistance force of the windwheel rotation depending on the generator loading, \( b \) is the distance between the windwheel axis and the point of the resistance force application. Force \( F^* \) was measured using a strain-gauge balance and the generator loading was adjusted by excitation current and the loading resistance.

Also, a generalized expression for the windwheel power \([3]\) reads

\[
N_w = \eta N_\infty = \eta \left( \rho V^3 / 2 \right) \left( \pi D^2 / 4 \right), \tag{3}
\]

where \( N_\infty \) is power of wind, \( \eta \) is the windwheel power coefficient with its maximum making \( \eta_{\text{max}} = 0.593 \), \( \rho \) is air density at the height of windwheel axis. Actual values of \( \eta \) depend on the parameters \( V, i, \lambda, C, G \) and \( \theta \), the latter being relative speed of the cylinders rotation

\[
\theta = \omega_c d / 2V = \pi n_c / 60V , \tag{4}
\]

with \( \omega_c \) as cycles per second and \( n_c \) as revolutions per minute.

Characteristics of the WT effectiveness include power coefficient of the windwheel \( \eta \), its high-speed \( Z \) and power losses at the cylinders rotation \( N_i \). High-speed of the windwheel (its relative speed of rotation) is given by the formula

\[
Z = \omega_c D / 2V = \pi D n_w / 60V . \tag{5}
\]

Power losses at the cylinders rotation, excluding those in the drive unit, make

\[
N_i = N_c + N_s + N_i , \tag{6}
\]

where \( N_c \) and \( N_s \) fall on the cylinders and the end plates, respectively \([4]\), while \( N_i \) stands for induced drag associated with the tip vortices. Taking into account results of \([5]\), an expression for the coefficient of induced drag reads

\[
\eta_i = N_i / qS = K_s C^2_s / \lambda . \tag{7}
\]

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**Fig. 2.** Wind-tunnel data on the windwheel speed of rotation at \( \lambda = 14, G = 4, i = 4 \) (a) and 6 (b), \( V = 1.5 \) (1), 2.2 (2), 2.8 (3) and 3.5 (4) m/s; optimum values of \( n_w^* \) and \( n_i^* \) (A).

**Fig. 3.** Optimum values of \( n_w^* \) (a) and \( n_i^* \) (b) vs. the oncoming-flow velocity \( V \) at \( \lambda = 14, G = 4, i = 4 \) (○) and 6 (●).
In this formula, \( q \) is dynamic pressure, \( S \) is the middle cross section of the cylinder and \( K_i \) is a coefficient depending on configuration of the cylinders, in this study we consider only the uniform ones.

**Experimental results.** Fig. 2 shows wind-tunnel data on \( n_w(n_c, V) \) obtained at \( D = 2.0 \) m, \( V = 1.5 \div 3.5 \) m/s, \( \lambda = 14 \), \( G = 4 \), \( i = 4 \) (a) and \( i = 6 \) (b). It is seen that functions \( n_w(n_c) \) come to their maximum (optimum) values \( n_w^* \) at \( n_c = n_c^* \), curve A in the figure. Comparing the cases of \( i = 4 \) and 6, one can observe that \( n_w^* \) is higher in the second one, the difference making 2 per cent at \( V = 3.5 \) m/s and 15 per cent at \( V = 1.5 \) m/s. At the same time, \( n_c^* \) goes down with increase of \( i \) and diminution of \( V \). That is, changing of \( i \) from 4 to 6 reduces \( n_c^* \) by 10 per cent at \( V = 2.2 \) m/s and by 20 per cent at \( V = 3.5 \) m/s.

Variations of \( n_c^*(V) \) and \( n_w^*(V) \) corresponding to Fig. 2 are given in Fig. 3. These are linear functions which can be extrapolated to higher wind velocities up to \( V = 8 \div 9 \) m/s, whereas at \( V > 9 \) m/s the behavior of windwheel characteristics becomes different. Here \( V_0 \) corresponds to the beginning of windwheel rotation and is reduced at growth of \( V \), \( \lambda \) and \( i \) so that the windwheel performance improves.

Fig. 4 shows \( \theta^*(V) \) and \( Z^*(V) \) due to expressions (4), (5) and linear extrapolation of \( n_c^*(V) \) and \( n_w^*(V) \) at \( V > 3.5 \) m/s. One can find that \( i = 6 \) is preferable with much lower values of \( \theta^* \), up to 40 per cent at \( V = 8 \) m/s, comparing to the case \( i = 4 \). In its turn, diminution of \( \theta \) reduces power losses at the cylinders rotation which are nearly proportional to \( i\theta^3/\lambda_3 \), see in [4]. At \( V = 8 \) m/s they drop by 30 per cent when changing from \( i = 4 \) to \( i = 6 \).

**Calculation of the WT characteristics.** A calculation method for prediction of the WT characteristics is based on the linearity of \( n_w^*(V) \). Fig. 5 illustrates the effect of cylinders aspect
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ratio upon the derivative $E = dn_w^*/dV$. Maximum values of $E$ corresponding to the highest windwheel power are observed at $\lambda = 14 \div 16$, recall that the optimum number of cylinders makes $i = 6$. In previous studies on the present subject, $\lambda = 6$ and $i = 3$ were tried reducing the windwheel effectiveness.

Approximating the experimental data on $E(\lambda, G, i)$ at $D = 1.9 \div 2\, m$, we obtain the following relations which are applicable at the cylinders aspect ratio at least up to $\lambda = 14 \div 16$:

\[
E = \frac{dn_w^*}{dV} = \left( a_0 + K_\lambda G^2 \right)^{-1},
\]

\[
Q = 0.55 \cdot 10^{-3} G^2 E^2,
\]

\[
\eta = 0.6Q \frac{(V - V_0)^2}{V^3}, \quad V \geq V_0,
\]

where in the case $i = 6$ it will be

\[
a_0 = 0.034, \quad K_\lambda = \left( \frac{9 \lambda^3}{\lambda} \right)^{-1}.
\]

For the windwheel high-speed $Z$ at $D = 1.9\, m$, expressions (1), (2) and (5) yield

\[
ZG = 5.5\sqrt{\eta V}.
\]

The windwheel power $N_w$ is determined according to (3) provided the values of $\eta$, $D$, $V$ and $\rho$, are known. Starting velocity $V_0$ in expression (10) and other relations is approximated as

\[
V_0 = 5.8\sqrt{\left( \frac{\lambda}{\lambda^2 - (1 - r_0)^2} \right)}, \quad \lambda = 10 \div 16,
\]

$r_0$ being the relative distance from the windwheel axis to the rotating cylinder.

Following to (9), Fig. 6 shows variations of $Q(i, G)$ where the maxima $G_M$ and $Q_M$ correspond to the highest power coefficient $\eta = 0.593$. At the minimum permissible values of these parameters, that is $G_m = 0.7G_M$ and $Q_m = 0.9Q_M$, we have $\eta = 0.53$ which is, taking into account power losses at the cylinders rotation, not lower than the maximum power coefficient of the blade WT$\theta$s at $V = 8\, m/s$. Thus, the optimum interval of $\eta$ for the Magnus WT$\theta$s makes $0.53 \div 0.59$. Taking into account $V_0$, the optimum number of cylinders will be $i = 6$.

Calculations of the windwheel characteristics start at the point $V = V_1$, where $\eta$ (10) crosses a line of optimum $\eta$ one can choose in the interval $\eta = 0.53 \div 0.59$. At $V_0 = 0$ we obtain

\[
V_1 = 0.022\frac{\lambda^2}{\eta_1}.
\]

To satisfy the condition $\eta \leq \eta_{\text{max}}$ in the range $V \leq V_1$ (with the exception of $V < 1 \div 2\, m/s$), the parameter $G$ should be corrected using the following relation coming from (12)

\[
G = 6.8G_1\sqrt{\eta_1 V / \lambda}.
\]

In particular, at $\lambda = 15$, $\eta_1 = 0.53$ and $G_1 = 6$, relation (15) gives $G = 2(V)^{1/2}$ and $Z = 2$, the latter is approximately twice lower than for most of the blade WT$\theta$s. In this range, the parameter $\theta$ keeps constant so that the speed of cylinders rotation $n_c$ is proportional to wind velocity.

Then, at $V_1 \leq V \leq V_2$, where $n_c = \text{const}$, the parameter $\theta$ is reduced with growth of $V$ and power losses at the cylinders rotation are diminished. In this case, for the power coefficient we have $\eta = \eta_1 V_1 / V$.

In the case of $V > V_2$ and $N_w = \text{const}$, the power coefficient is given by $\eta = \eta_2 (V_2 / V)^3$. In this range, the parameter $\theta$ and power losses at the cylinders rotation are also reduced. Relative speed of the cylinders rotation becomes lower with diminution of $r_0$ down to about 0.1, as well. Effect of $r_0$ upon $\eta$ at $\eta_1 = 0.53$ and 0.59 is illustrated in Fig. 7.

In Fig. 8 the behavior of $\eta(V)$ at $\eta_1 = 0.53$ is presented with and without taking into account power losses at the cylinders rotation, curves 1* and 1, respectively. Curve 2 corresponds to the coefficient $\eta$ of the blade WT$\theta$s. Wind velocity repetition in continental regions with the average annual $V = 4 \div 5\, m/s$ is shown by curve 3. It is seen that utilization of the Magnus WT$\theta$ is the most beneficial at $V < 8\, m/s$. 
Finally, we point out that the Magnus WT can be operated with the resultant power coefficient not smaller than $\eta = 0.43$ even at reduction of wind velocity to $V = 2$ m/s. Notice that for most of the blade wind turbines coefficient $\eta$ comes to zero at $V < 4$ m/s as far as their driving force goes down rapidly in this range of wind velocities. As a result, in the case of Magnus WT, power output is increased (2.6 to 1.2 times at $V = 5 \div 7$ m/s) with its daily production up to 20 hours and more. Also, the upper limit of wind velocity for the Magnus WTs may be as high as 35 to 40 m/s instead of about 25 m/s for the blade turbines.

**Conclusions.** Through wind-tunnel tests of the Magnus WT model with rotating cylinders and calculations of the windwheel characteristics, optimal parameters of the windwheel were determined testifying to its benefits comparing to the blade wind turbines. Thus, it is found that the most appropriate for the WT design is the number of rotating cylinders equal to 6 with their aspect ratio of 15. Most of all, the Magnus WTs are preferable in the range of wind velocity $V < 8$ m/s where the windwheel power coefficient and wind velocity repetition are close to their maxima, whereas the blade turbines are much less effective. This feature results in extended daily operation of the Magnus WTs and increased power production. Moreover, the windwheel speed of rotation can be reduced two times comparing to the blade turbines, ensuring ecological and operational safety of the Magnus WTs.

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**REFERENCES**