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Numerical investigations of transition between regular and Mach reflections caused by free-stream disturbances

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Abstract. Numerical simulations have been performed to study the influence of the free-stream disturbances on the alternation of the steady shock wave reflection configurations in the dual solution domain. Different types of disturbances have been considered. The analysis of interaction between disturbances and the incident shock wave can be substantially simplified for the localized density disturbances. It is shown that such disturbances can indeed cause the transition from regular reflection to Mach reflection and back, so that within a certain range of angles of incidence the shock wave reflection configuration can be considered as a bi-stable system. The threshold amplitude of the localized density disturbance, able to induce the transition, has been estimated theoretically. The results of numerical computations convince of higher stability of the Mach reflection in the dual solution domain compared to the regular reflection, which is in accordance with available experimental data.

Key words: Regular reflection, Mach reflection, Hysteresis phenomenon, Euler numerical simulation, Free-stream disturbances

1 Introduction

The reflection of shock waves is a problem of fundamental interest in supersonic gas dynamics. It is well known (Hornung 1986, Ben-Dor 1991) that for the steady reflection there exist only two possible configurations: the regular reflection and the Mach reflection. The criteria of transition between them were a subject of discussion for several decades. John von Neumann in a famous paper (von Neumann 1943) put forward two different criteria known today as the detachment criterion and the mechanical equilibrium (or von Neumann) criterion. The former gives the maximum angle of incidence \( \alpha_d \), at which the regular reflection is still possible theoretically, while the latter determines the minimal angle \( \alpha_N \) for existence of the Mach reflection. The angle \( \alpha_N \) can be introduced only if the flow Mach number \( M \) exceeds a certain value, which for air (more exactly, for a perfect gas with the specific heat ratio \( \gamma = 1.4 \)) is equal to approximately 2.2 (see, for example, Hornung 1986, Ben-Dor 1991). For \( M > 2.2 \) there exists the range of angles of incidence \( \alpha_N < \alpha < \alpha_d \) (so called dual solution domain) where both regular reflection and Mach reflection of steady shock waves are theoretically possible – see Fig. 1.

The existence of the dual solution domain implies a possibility of hysteresis phenomena at the transition. Hornung et al. (1979) proposed that such phenomena could be observed experimentally if the shock wave angle increases and then decreases smoothly during the experiments. They assumed that in this case the transition from regular to Mach reflection should occur when \( \alpha \) reaches \( \alpha_d \). At this angle value, the Mach stem emerges and grows abruptly to its full size. If now the angle is decreased the Mach reflection should persist throughout the dual solution domain down to \( \alpha_N \) where the reverse transition must happen. The Mach stem height decreases continuously with decreasing \( \alpha \) and vanishes at \( \alpha = \alpha_N \).

To confirm this hypothesis, Hornung and Robinson (1982) carried out a wind tunnel experiment and observed no hysteresis: both transitions, from regular to Mach reflection and from Mach to regular reflection, occurred at the same value of shock wave angle which was very close to \( \alpha_N \). They explained this result by the instability of regular reflection above \( \alpha_N \) with respect to flow disturbances existing in any experimental facility. More than ten years later, however, the predicted hysteresis was observed almost simultaneously in numerical simulations (Ivanov et al. 1995) and experiments (Chpoun et al. 1995).

Further numerous computational works (see, for instance, Chpoun and Ben-Dor 1995, Ivanov et al. 1996, Hadjadj et al. 1998, Ivanov et al. 1998a) confirmed the existence of the hysteresis phenomenon with the transi-
tion angles in close agreement with the predictions of Hornung et al. (1979). Experimental investigations, however, were more ambiguous. In the experiment of Chpoun et al. (1995) performed at free-stream Mach number \(M = 4.96\), the transition to Mach reflection was observed at \(\alpha = 37.2^\circ\), which is in the dual solution domain noticeably lower than \(\alpha_d = 39.2^\circ\). It should be noted that, in contrast to the original experiment of Hornung and Robinson (1982), these tests were performed in an open jet facility rather than in a wind tunnel with a closed test section. Later, Fomin et al. (1996) and Ivanov et al. (1998b) conducted new experiments in both types of wind tunnels. No hysteresis at all or a very small (fractions of a degree) hysteresis loop were observed in the wind tunnel with a closed test section while the difference between the angles of direct and reversed transition in the open jet facility was approximately 3–4° at \(M = 6\). In experiments performed by Sudani et al. (1999) in a wind tunnel with a closed test section the hysteresis loop of a few degrees was observed at \(M = 4\).

Several issues may be important when considering the reasons for the discrepancies existing between the results obtained at different wind tunnels. The wind tunnels differ in type of the test section (closed or open-jet) and its shape (round or rectangular). The size of the test section, or, more exactly, relative sizes of the model and the test section may also be crucial for the transition, because the farther is the model from the wind tunnel walls the less noisy is the flow in the shock wave reflection area. Spanwise aspect ratio of the model may also be important because of the flow three-dimensionality, which may affect the transition. The most important is that different wind tunnels certainly have different levels of free stream disturbances, which could promote the transition from regular to Mach reflection.

An explanation of all these discrepancies was proposed in Ivanov et al. (1998b) where the conclusion of Hornung and Robinson (1982) concerning an instability of regular reflection was modified as follows. Both regular and Mach reflections are stable within the dual solution domain with respect to small disturbances. However, regular reflection is unstable to disturbances whose amplitude exceeds a certain threshold level. Naturally, the threshold amplitude must decrease when \(\alpha\) increases and vanish at \(\alpha = \alpha_d\). Thus, in numerical simulations where any free-stream disturbances are absent, it is possible to maintain regular reflection just up to \(\alpha = \alpha_d\). On the other hand, in experiments the maximum angle, at which regular reflection can still be observed, probably depends on the level of disturbances inherent in the experimental facility used.

In recent experiments performed by Ivanov et al. (2001) in a low-noise wind tunnel with a closed test section the hysteresis has been observed in close agreement with theoretical predictions. Nevertheless, the influence of free-stream disturbances on the transition between regular and Mach reflections has not been yet clarified. The hypothesis put forward by Ivanov et al. (1998b) could certainly be justified if one would be able to demonstrate in numerical simulations that disturbances can actually trigger the transition from regular to Mach reflection, and also that the type and the amplitude of those disturbances are relevant to real experimental facilities. It is believed that, should this study be made, the long history of the attempts to solve the problem of shock wave reflection transition in steady flows would be close to its completion.

Up to now, the transition between regular and Mach reflections triggered by disturbances has been investigated in several works. Ivanov et al. (1997, 1998a, 1998c) focused on disturbances in the form of strong short-time changes in the free-stream velocity. It was found that, disturbing the flow in such a way, it was possible to force the transition from regular to Mach reflection. However, such a kind of disturbances result in a system of traveling gas dynamic discontinuities, which interact with the shock wave system in a complex way, and it is not easy to analyze their influence. In addition, the amplitude of the investigated disturbances was very high. In the present paper we investigate the influence of disturbances on the transition in a more systematic manner. The free-stream disturbances of three elementary types existing in compressible fluid, i.e. shock waves, expansion waves and contact discontinuities, are considered. One particular kind of the disturbance of the last type, localized disturbances of density, proves to be especially convenient for investigation of shock wave reflection transition. Using such disturbances, we demonstrate that the flow containing a reflecting steady shock wave can be actually considered within the dual solution domain as a “bi-stable system”. There are two steady states, regular and Mach reflections, stable to small disturbances. However, the transition from one state to another can be caused by finite-amplitude disturbances of a certain kind and amplitude. Further, it is much easier to cause the transition from regular to Mach reflection than the reverse one. For the latter, the disturbances of larger amplitude and more length in both space and time are necessary. A simple theoretical considera-
tion is given for predicting the amplitude of disturbance sufficient to induce the transition.

The paper is organized as follows. In the next section, the details of the numerical method and flow conditions are described. The numerical results concerning the influence of disturbances on the shock wave reflection transition are presented and discussed in Sect. 3. Afterwards, in the last section, the conclusions are formulated.

2 Problem formulation and numerical techniques

The two-dimensional time dependent Euler equations are written as follows:

$$\begin{align*}
\partial Q/\partial t + \partial F/\partial x + \partial G/\partial y &= 0, \\
Q &= \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ p \end{pmatrix}, \\
F &= \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u^2c \end{pmatrix}, \\
G &= \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v^2c \end{pmatrix}.
\end{align*}$$

Here $p$ is the density, $e$ is the total energy per unit volume, $\rho$ is the pressure, $u$ and $v$ are the velocity components in the $x$ and $y$ directions, respectively. The equation of state is

$$p = (\gamma - 1)\left(e - \rho \frac{u^2 + v^2}{2}\right),$$

where $\gamma = 1.4$ is the specific heats ratio.

The HLLE (Harten-Lax-van Leer-Einfeldt) solver (Einfeldt et al. 1991) is used to calculate the numerical fluxes on the inter-cell boundaries because of its robustness for flows with strong shock waves and expansions. The subscripts $L$ and $R$ correspond to the states on the left-hand and right-hand sides of the boundary between the grid cells $i, j$ and $i + 1, j$, respectively; then, omitting the subscripts $j$, the numerical flux through the boundary $\hat{F}_{i+1/2}$ given by the HLLE solver is

$$\begin{align*}
\hat{F}_{i+1/2} &= \frac{a^+F^L - a^-F^R + a^+a^-\left(Q^R - Q^L\right)}{a^+ - a^-}, \\
a^- &= \min\{0, u_n^L - c^L, \tilde{u}_n - \tilde{c}\}, \\
a^+ &= \max\{0, u_n^R + c^R, \tilde{u}_n + \tilde{c}\}.
\end{align*}$$

Here $u_n$ is the normal-to-the-boundary component of the velocity, $c = \sqrt{\gamma p/\rho}$ is the speed of sound, and the tilde marks quantities which are obtained by Roe averaging between the states $Q^L$ and $Q^R$. The variables in the states $Q^L$ and $Q^R$ are reconstructed from cell averaging using the 4-th order formula (Yamamoto and Daiguji 1993).

$$\begin{align*}
q^L &= q_i + (Dq_{i+1/2} + 2Dq_{i+1}/6), \\
q^R &= q_{i+1} - (Dq_{i+1/2} + 2Dq_{i+1}/6), \\
Dq_{i+1/2} &= m(Dq_{i+1/2}, bDq_{i+1/2}), \\
Dq_{i+1} &= m(Dq_{i+1}, bDq_{i+1}).
\end{align*}$$

Fig. 2. Schematic of the flow

$$\begin{align*}
Dq_{i+1/2} &= \Delta q_{i+1/2} - \Delta^3 q_{i+1/2}/6, \\
\Delta^3 q_{i+1/2} &= \Delta^2 q_{i-1/2} - 2\Delta q_{i+1/2} + \Delta^2 q_{i+3/2}, \\
\Delta^2 q_{i+1/2} &= m(\Delta q_{i-1/2}, b_1 \Delta q_{i+1/2}, b_1 \Delta q_{i+3/2}), \\
\Delta q_{i+1/2} &= m(\Delta q_{i+1/2}, b_1 \Delta q_{i-1/2}, b_1 \Delta q_{i+3/2}).
\end{align*}$$

Here $\Delta q_{i+1/2} = q_{i+1} - q_i$ and the minmod function $m$ is determined as

$$m(x_1, \ldots, x_n) = \begin{cases} s \min(|x_1|, \ldots, |x_n|), & if \ \text{sgn}(x_1) = \ldots = \text{sgn}(x_n) = s \\ 0, & otherwise. \end{cases}$$

The parameters $b, b_1$ are set $b = 3, b_1 = 2$ in our computations. The reconstruction is applied to the primitive variables $\mathbf{q} = (\rho, u, v, p)^T$. The use of a high-order reconstruction formula allows to decrease a large numerical diffusion inherent to the HLLE solver and provide a high resolution of the smooth part of the solution without losing robustness near strong shock waves. The third-order explicit Runge-Kutta scheme (Shu and Osher 1988) is chosen for advancing the solution in time.

A sketch of the flow and the computational domain is given in Fig. 2. The wedge, which is a right-angled triangle with a 15° leading edge angle (as in the experiments of Fomin et al. 1996 and Ivanov et al. 1998b), is installed in a uniform supersonic stream. The angle of attack $\theta$ and, correspondingly, the angle of the incident shock wave $\alpha$ are varied by rotating the wedge. The distance between the trailing edge and the lower boundary of the computational domain is $g = 0.42w$ where $w$ is the wedge chord. The left boundary is a supersonic inflow where all variables are specified. At the lower boundary we use symmetry boundary conditions, which is in accordance with the experiments where two symmetrical wedges are usually used in order to eliminate the influence of the boundary layer. The right boundary of the domain is located far enough downstream so that the flow is supersonic across it. A multi-block quadrilateral structured grid with a total number of cells of about 90,000 is used in the computations. The simulations of the influence of the disturbances
on shock wave reflection were performed in the following way. Initially, we obtain a steady-state shock wave reflection configuration, either of regular or Mach type. Then the flow on the left boundary is disturbed and this change in the inflow boundary conditions is maintained for some time $\Delta t$. After that, the disturbance is “switched off” and all quantities on the left boundary return to their undisturbed values. More detailed description of different kinds of disturbances used will be given below.

All computations were performed at the free-stream Mach number $M_\infty = 4$. At this Mach number $\alpha_N = 33.4^\circ$ and $\alpha_U = 39.2^\circ$.

### 3 Results

It is well known that there are three types of elementary waves in a compressible fluid: shock waves, expansion (rarefaction) waves and contact discontinuities. The two former types can be referred to as pressure waves, because the pressure is changed across them. On the contrary, the pressure is continuous across a contact discontinuity, but the density, the entropy and the tangential velocity can be discontinuous. If some variation of the free stream values takes place (they are changed from constant values) the emerging discontinuity turns into a combination of these elementary waves travelling with different velocities. The middle wave is a contact discontinuity while the leftmost and rightmost waves may be either shock or expansion waves. Thus, it seems reasonable to investigate the effect of each of these types of disturbances on the shock wave reflection transition.

#### 3.1 Transition caused by pressure wave disturbances

For both types of pressure waves there are two different families of solutions, left-running and right-running waves with respect to the fluid. Thereafter, we will refer to these waves as left-running and right-running shock and expansion waves but it should be kept in mind that in our case, for a high Mach number supersonic flow, both of them travel downstream in the laboratory frame of reference.

If we consider the theoretical criteria of transition re-plotted in the $(\theta, M)$-plane where $\theta$ is the wedge angle of attack and $M$ is the flow Mach number (see Fig. 3), it can be expected that decreasing the flow Mach number at fixed $\theta$ may be favorable for inducing the transition to Mach reflection.

The Mach number is changed behind the pressure wave disturbance to the value $M_{\text{dist}}$. Given the amplitude of the pressure jump, $M_{\text{dist}}$ may be easily evaluated using either Rankine—Hugoniot relations in case of a shock wave or isentropic relations in case of an expansion wave. The dependence of $M_{\text{dist}}$ on the pressure change across a pressure wave is shown in Fig. 4. It is obvious that the Mach number can be decreased most efficiently in a left-running expansion wave.

The results of numerical experiments with a left-running expansion wave are given in Fig. 5. The simulation was performed for the angle of the incident shock chosen approximately in the middle of the dual solution domain, $\alpha = 37^\circ$ that corresponds for $M = 4$ to the wedge angle of attack $\theta = 23.83^\circ$. The expansion wave disturbance was introduced on the left boundary. The forward front of the expansion wave moves at the velocity $U_\infty - c_\infty$ ($U_\infty$ and $c_\infty$ are, respectively, the fluid velocity and the speed of sound in the undisturbed stream). The pressure upstream of the expansion wave $p_{\text{dist}} = p_\infty + \Delta p$ was two times higher than the undisturbed pressure $p_\infty$, so that $\Delta p/p_\infty = 1$.

In the initial phase the shock wave configuration was a steady regular reflection (Fig. 5a). Figure 5b shows the interaction between the propagating expansion wave and the steady oblique shock. The Mach number behind the expansion wave is approximately 3.15, and a regular reflection cannot exist at such parameters – see Fig. 3. As
a result, the transition to a Mach reflection occurs at $t = 1.15$ (thereafter $u/c_\infty$ is used as a time unit). After this moment the disturbance was “switched off”. Restoring the flow parameters to their original values generates a weak contact discontinuity and a terminating shock wave moving from the left boundary – Fig. 5c. The interaction with this rear shock leads to the emergence of a “broken” shock wave (see Fig. 5d) whose angle in the vicinity of the triple point is substantially greater than at the wedge leading edge. Such a phenomenon at moving shock/steady oblique shock interaction was mentioned earlier in Ivanov et al. (1998a). It emphasizes the growth of the Mach stem. Another interesting feature is formation of an intense vortex behind the Mach stem (Fig. 5e). It convects downstream and leaves the computational domain. After that, the Mach stem continues its slow growth and finally the steady state Mach reflection shown in Fig. 5f is formed.

In this numerical experiment the duration of the disturbance, i.e. the time period between the start of disturbing the flow and restoring the flow parameters to the original free-stream values, $\Delta t = 1.15$, was sufficient for the transition to a Mach reflection to occur during this period. However, it was possible to decrease $\Delta t$ substantially, i.e. to decrease the gap between the moving expansion wave and a terminating rear shock. We were able to observe the transition when $\Delta t$ was as small as 0.05. It seems that in the latter case, the transition is connected mostly with the above-mentioned phenomenon of increasing of the angle of incident shock after its interaction with the terminating shock wave of the disturbance.

The situation when the pressure wave disturbance consisted of a “left-running” shock wave followed by an expansion wave was also simulated. In this case $\Delta p/p_\infty$ was equal to $-0.5$. In spite of increasing the Mach number upstream of the moving shock wave up to 4.96, the transition to Mach reflection was also observed. The reason for this was also the “broken” shock wave resulted from the interaction. However, the attempts to decrease noticeably the amplitude of disturbance were not successful: so, the disturbance with $\Delta p/p_\infty = -0.25$ could not force the transition.

It should be noted that we were not able to induce the reverse transition from Mach to regular reflection using all the above-described disturbances of a reasonable level. Probably, it can be explained taking into account the dependence of $\theta_N$ on the Mach number (Fig. 3). There is the maximum value of $\theta_N$ equal to $\theta_{N,max} = 20.92^\circ$ which is reached at $M = 4.46$. If the wedge angle (as in our case) exceeds $\theta_{N,max}$ no variation of the Mach number can create the state for which only a regular reflection is possible. Of course, this argument is based on a quasi-steady state assumption and it cannot be entirely applicable to real dynamical process of the interaction between a disturbance and a shock reflection configuration. Nevertheless, one can conclude that a Mach reflection is a more stable configuration than a regular reflection.

### 3.2 Transition caused by contact discontinuity disturbances

Another possible type of disturbances is the contact discontinuities. In this case, the Mach number behind a disturbance is $M_{dist} = M_\infty \sqrt{1 + \Delta p/p_\infty}$ because the pressure and the velocity were not changed. Figure 6 shows the interaction between a contact discontinuity disturbance and a steady shock wave for $\alpha = 37^\circ$, $\Delta p/p_\infty = -0.25$ and $\Delta t = 1$.

Behind the first contact discontinuity (see Fig. 6a) the flow Mach number decreases to 3.46, which is close to the value on the detachment criterion line in Fig. 3. The second contact discontinuity is generated at the time $t = \Delta t$ when the disturbance is switched off. Its interaction with the incident shock wave (Fig. 6c) slightly increases the slope of the shock wave. It proves to be sufficient for the transition and Mach reflection emerges immediately after the time when the contact discontinuity passes the reflection point (Fig. 6d). Then the Mach stem height grows (Fig. 6e) and finally the flow reaches the steady state (Fig. 6f).
All disturbances considered above are travelling plane waves. They interact with the incident oblique shock wave not only in the vicinity of the reflection point but along all its length. However, it is also possible to introduce a disturbance in some small 2D region without disturbing the entire flow. If the density in this region is changed, but the values of pressure and velocity coincide with those of the surrounding uniform flow, then such a “spot” of density (and of temperature, entropy, Mach number) simply convects downstream, being separated from the undisturbed flow by a contact surface. We can introduce a localized density disturbance upstream of the reflection point and the interaction with steady wave configuration starts when it reaches this point.

The example of such interaction is given in Fig. 7. The computations were performed for $\alpha = 36^\circ$ that corresponds to the wedge angle of attack $\theta = 23.01^\circ$.

It was started from the uniform $M_\infty = 4$ flow initial conditions. The steady state regular configuration, shown in Fig. 7a, was obtained. Then, in order to trigger the transition to Mach reflection, a density disturbance with an amplitude of $\Delta \rho / \rho_\infty = -0.25$ was introduced at the inflow boundary in 10 lower cells (the thickness of the disturbed layer was $\approx 0.05w$). The choice of the level of the disturbance will be discussed below. The thickness of the disturbed layer was varied in different simulations. The results presented below were obtained with the minimal thickness required for the flip from one type of reflection to the other. Figure 7b demonstrates the flow immediately after the transition, when the non-dimensional time from the beginning of the disturbance is $t = 0.6$. The density disturbance is seen in a narrow stripe near the symmetry plane. A developing Mach stem is clearly seen near the reflection point. The disturbance was “switched off” after that time, and the Mach stem persisted and continued to grow. Finally, we obtained a steady-state Mach reflection, which is shown in Fig. 8a.

The results of simulation of the forced reverse transition from Mach to regular reflection are given in Fig. 8. In this case, the density was increased in 20 lower cells (the thickness of the disturbed layer $\approx 0.1w$) by $\Delta \rho / \rho_\infty = 0.6$ so that the triple point of the Mach reflection configuration was within the disturbed zone. As a result of interaction of the disturbance with the triple shock configuration, the Mach stem height starts decreasing, and finally the Mach stem disappears. This is shown in Fig. 8b, which corresponds to the non-dimensional time $t = 2.5$ from the start of perturbing the flow. At this time, the disturbance was already “switched off” and its “tail” is seen upstream from the reflection point. Note that the time necessary for the transition from Mach to regular reflection appears to be significantly longer than that for the transition to Mach reflection. After the disturbance had been switched off, the resulted regular reflection configuration maintained, and finally, we obtained the flow field identical to that in Fig. 7a.

The physical mechanism, which causes the change in the shock wave reflection configuration, is essentially the same in both cases. The incident shock wave entering the disturbed layer with a different density (and, correspondingly, Mach number) is refracted and impinges on the symmetry plane at a different angle. In the first case, the density was decreased and the shock wave angle at the reflection point was larger than the detachment criterion angle determined for the perturbed layer parameters. This forced the transition to Mach reflection. In the second case, on the contrary, the density was increased and the angle of the refracted shock became smaller than the von Neumann angle for the perturbed layer parameters. As a result, Mach reflection could not exist any longer and the transition to regular reflection occurred.

The angle of the refracted shock $\alpha_{rs}$ can be easily determined analytically by solving the spatial Riemann problem (see Fig. 9) between two states: the flow after the incident shock (1), and the flow in the disturbed layer (2). The solution can be obtained using the pressure/deflection angle diagrams of shock and rarefaction waves. For the
Fig. 7a,b. Transition from regular to Mach reflection. Incident shock wave angle $\alpha = 36^\circ$. Level of the disturbance $\Delta \rho = -0.25 \rho_\infty$. Density contours

Fig. 8a,b. Transition from Mach to regular reflection. Incident shock wave angle $\alpha = 36^\circ$. Level of the disturbance $\Delta \rho = 0.6 \rho_\infty$. Density contours

Fig. 9. Refraction of incident shock wave

Fig. 10. Pressure/deflection angle diagrams for reflection of shock wave on contact discontinuity (slip surface)

From which the angle of the refracted shock wave $\alpha_{rs}$ can be easily evaluated.
ary condition downstream (see Salas and Morgan 1983 for discussion). However, there is some evidence that a strong steady shock wave can be maintained in asymmetrical shock wave reflection, where the inherent mechanism exists for providing necessary pressure behind the strong shock (see Li et al. 1999, Khotyanovsky et al. 2001).

We can now directly evaluate the level of the density disturbances needed for both, regular to Mach and Mach to regular, reflection transitions. These threshold levels of $\Delta \rho$ can be determined from the condition that $\alpha_{rs}$ is equal to $\alpha_d$ or to $\alpha_N$, respectively. They are given in Fig. 11 vs the incident shock wave angle $\alpha$. It is apparent that, for example, at $\alpha = 36^\circ$ a change in density by $\Delta \rho = -0.25\rho_\infty$ is sufficient for the transition from regular to Mach reflection to occur, and $\Delta \rho = -0.15\rho_\infty$ is not, which was confirmed in our computations. At the same time, at $\alpha = 38^\circ$ $\Delta \rho = -0.15\rho_\infty$ is quite enough for the transition to Mach reflection, which is documented in Fig. 12 where the moment just after the transition $t = 0.6$ (Fig. 12a) and the final Mach configuration (Fig. 12b) are shown.

It is apparent from Fig. 11 that the slope of the line corresponding to the transition to regular reflection is much higher than that for the transition to Mach reflection. Consequently, the levels of the disturbance capable of changing the Mach configuration into the regular one are very high and comparable in magnitude with free-stream density in some range of $\alpha$. Such a behaviour could be expected, bearing in mind the change of the Mach number in the disturbed layer and a strong dependence of $\alpha_N$ on the Mach number (see Fig. 1).

There is another reason for making the transition from regular to Mach reflection much easier than the reverse transition. For the transition to Mach reflection it is enough to have a disturbance localized in a very small region near the reflection point. The disturbance capable of changing MR into RR must have much larger scale both in space and in time. As it is mentioned above, its spatial extent must exceed the Mach stem height. The duration of the disturbance must be long enough so that the relatively slow process of decreasing of the Mach stem could be completed.

All these facts allow us to conclude that the Mach reflection configuration is in some sense more stable in the dual solution domain than the regular one. This is in agreement with the results of the previously cited experiments, where the transition to regular reflection always occurs at the angles close to $\alpha_N$ and usually slightly higher than $\alpha_N$. It is important to notice that we do not consider the disturbances of the type used in this study to be mainly responsible for the early transition to Mach reflection observed in the experiments. We regard them only as a useful tool for studying the possibility of the forced transition from one reflection type to the other and the conditions of such transition.

4 Conclusions

The possibility of the forced transition between regular and Mach reflection in the dual solution domain by means of free-stream disturbances is shown numerically. Consequently, within the dual solution domain, the shock wave reflection configuration can be considered as a bi-stable system. For some kind of disturbances, localized density disturbances, the threshold level of the disturbances capable of causing the transition may be evaluated by a simple theoretical analysis. It is found that the transition from regular to Mach reflection is much easier to perform than
the reverse transition; therefore, the Mach reflection configuration can be regarded in accordance with the experimental observations as more stable in the dual solution domain compared with the regular one.

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