NUMERICAL INVESTIGATION OF SHOCK WAVE INTERACTIONS IN SUPERSONIC IMPERFECTLY EXPANDED JETS

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Introduction

It is well known that the aerodynamic study of supersonic jets exhausting from convergent-divergent nozzles is a problem of great importance in many space and aeronautical applications. Various physical phenomena involved in this problem are directly linked to the performance of jet engines. Though off-design operations with either overexpanded or underexpanded exhaust flow induce performance losses, in many cases such regimes cannot be avoided. The imperfect matching between the ambient pressure and the exit nozzle pressure leads to the formation of a complicated shock wave structure. Passing through the system of shock waves, the flow gradually adapts to the ambient conditions. For several decades, numerous experimental, numerical, and analytical investigations of the structure of supersonic jets have been undertaken, but the subject is quite complicated and not yet clearly understood.

In recent years, significant progress was achieved in our understanding of fundamental aspects of the transition between regular and Mach shock wave reflections. It was revealed, both numerically \cite{1} and experimentally \cite{2}, that such a transition is accompanied by a hysteresis. Namely, if the incidence angle of the shock wave is varying, the transition to Mach reflection and the back transition to regular reflection are observed at different angles, as it was earlier conjectured by Hornung et al. \cite{3}. Now it is well documented \cite{4} that the transition to Mach reflection occurs at the incident shock wave angle $\alpha$ equal to the so-called detachment angle $\alpha_d$ and the back transition – at $\alpha$ equal to the von Neumann angle $\alpha_N$. The angles $\alpha_d$ and $\alpha_N$ are the theoretical criteria deduced from the analysis of shock wave reflection using pressure-deflection diagrams. Regular reflection is theoretically impossible above $\alpha_d$, whereas Mach reflection is not possible below $\alpha_N$. At high Mach numbers, these two angles bound an interval of incident shock wave angles (the dual solution domain), where both regular and Mach reflections can exist. Numerical experiments give clear evidence that the transition to Mach reflection occurs when, at increasing the angle, the upper boundary of the domain is crossed and the reverse transition is observed when, at decreasing the angle $\alpha$, the lower boundary is reached. Thus, two different types of reflection can be obtained at the same angle within the dual solution domain, and the change in the shock wave configuration is accompanied by a hysteresis phenomenon.

This conclusion was deduced from the study of a supersonic flow around two symmetrical wedges, which was used in most part of experimental studies and computations of shock wave reflection transition in steady flows. The reflection transition is also a salient feature of shock wave interactions in supersonic imperfectly expanded jets. The variation of the pressure ratio between the jet and ambient space changes the incidence angle of either the nozzle lip shock (for overexpanded conditions) or the barrel shock (for underexpanded jets). It can be assumed that the hysteresis phenomenon should also occur in such flows. The simplest geometrical configuration for this case corresponds to a plane overexpanded jet. Here, in contrast to axisymmetric jets and a plane underexpanded jet, both the incident shock wave and the jet boundary are straight (the latter before its interaction with the reflected shock wave). Hence, a comparison with the theoretical criteria can be made directly.

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Note, there exist some experimental [5, 6] and numerical [7] evidence of the hysteresis phenomena in underexpanded jets, however, it is difficult to say if they refer to the same type of hysteresis.

The goal of the present paper is to investigate numerically the shock wave reflection transition in a plane overexpanded jet in order to establish definitely whether the transition is accompanied by the hysteresis. At first, the Euler simulations are carried out with a high-order WENO (Weighted Essentially Non-Oscillatory) scheme. Further, a second order TVD (total variation diminishing) scheme is used to conduct Navier – Stokes computations with the $k$-$\varepsilon$ turbulence model. It allows us to investigate a more realistic case of a high-Reynolds-number turbulent jet and try to elucidate the impact of viscosity and turbulence on the shock wave reflection transition.

Problem formulation and numerical method

Euler computations. Shock wave interaction is primarily an inviscid phenomenon, and the attempt to reproduce it on the basis of the Euler equations seems to be natural. For this purpose, the 5th order finite difference WENO scheme [8] was utilized. WENO schemes are very appropriate for the problem under consideration because they have the property of robust shock capturing and provide high accuracy in the regions where the solution is smooth. The global Lax-Friedrichs splitting was applied when calculating numerical fluxes, and the 3rd order TVD Runge – Kutta scheme was used to advance the solution in time.

The nozzle exit was taken as a part of the left boundary of the computational domain. All quantities were fixed at the nozzle exit prescribing a uniform supersonic flow with the jet Mach number $M_j = 5$ and the stagnation temperature equal to that of the ambient fluid. The jet pressure $p_j$ was varied in order to change the angle of the nozzle lip shock $\alpha$. The remaining part of the left boundary was treated as a solid wall, and the reflection procedure was used to specify the variables in the ghost cells outside the computational domain. On the far field (upper) boundary, the conditions based on the Riemann invariants with a prescribed ambient pressure were used. "Soft" boundary conditions were imposed on the right (outflow) boundary, the streamwise derivatives of all quantities being put to zero. Owing to the flow symmetry, only a half of the real jet flow was computed, and the lower boundary was treated as a symmetry line. The grid cell size in all computations presented below was such that 100 cells were located across the nozzle half-width $h/2$.

Navier – Stokes computations. As well as in other free shear flows, the transition to turbulence occurs in jets at quite low Reynolds numbers. Thus, though the molecular viscosity can be considered as negligible in many practical applications, the turbulent, eddy, viscosity can play an important role. To elucidate the effect of turbulence on the transition between different types of shock wave reflection, Navier – Stokes computations with the $k$-$\varepsilon$ turbulence model were conducted. The Favre-averaged Navier – Stokes equations were integrated using an explicit second-order finite volume scheme. For the convective terms, the upwind TVD scheme of Harten and Yee [9] with van Leer’s flux limiter was used, and central-difference methods were employed for the diffusion terms of the momentum, energy, and turbulence equations. A second-order Runge – Kutta scheme was used for time marching.

The details of the turbulence model used can be summarised as follows. The $k$-$\varepsilon$ model is the most widely known and extensively used two-equation eddy viscosity model. It was originally developed to improve the mixing-length model and to avoid the algebraic description of the turbulent length scale in complex flows [10]. When computing turbulent free jet flows, the use of the standard $k$-$\varepsilon$ model is an appropriate way to correctly predict the mean and the turbulent flow parameters. However, for high-speed flows, it is important to include compressible dissipation and pressure-dilatation effects in the two-equation turbulence models.
as suggested by Sarkar et al. [11] and Vandromme [12]. In this study, we used an improved version of the $k$-$\varepsilon$ turbulence model to account for compressibility effects [13, 14].

The computations were performed using 10 processors on a parallel computer (ORIGIN-2000). The average computational time for each simulation was approximately 30 h CPU (global time).

The free-stream conditions were: $M_\infty \approx 0$, $p_\infty = 101322$ Pa, $T_\infty = 300$ K. Low free-stream values (nearly zero) of the turbulent kinetic energy $k$ and the dissipation rate $\varepsilon$ were fixed near the free-stream at rest. These values were kept constant for all the simulations. For turbulent free-shear flows, the $k$-$\varepsilon$ model showed no sensitivity to free-stream turbulence [15].

As well as in inviscid simulations, symmetry boundary conditions were applied along the axis of symmetry. Non-reflective boundary conditions with a fixed value of the static pressure were used along the outer boundary corresponding to the external free stream. For the turbulence transport equations, either zero-order extrapolation or free-stream values were used for $k$ and $\varepsilon$ along the outer boundaries. If the flow was outgoing along the outer boundary, zeroth-order extrapolation was used. If there was flow entrainment, then free-stream values were imposed along the outer boundaries. At the inflow, the nondimensional turbulent kinetic energy was defined as $K^* \equiv k^{1/2}/U_j$, where $U_j$ is the velocity of the jet. Once $k$ was known, $\varepsilon$ was obtained using the production-equals-dissipation hypothesis.

The size of the computational domain was $L_x = 2h$ in the streamwise direction and $L_y = h$ in the cross-streamwise direction. The computations employed 500x250 equally spaced points. Grid-independent results were obtained using this mesh; the height of the Mach stem was used as a criterion to obtain a grid-independent solution for $M_j = 5$ and $\alpha = 42^\circ$.

**Results**

**Inviscid computations.** The computations were started for the jet/ambient pressure ratio $p_j/p_a$ corresponding to the incident shock angle $\alpha = 41^\circ$. It is noticeably higher than the detachment angle, which is equal to $\alpha_d = 39.3^\circ$ for $M_j = 5$. Consequently, Mach reflection is only possible in this case. The ambient conditions in the entire computational domain were taken as the initial flowfield for this computation. After the complex transient process of start-up of the jet flow, the flow began evolving to the steady state, and a converged solution was finally achieved. It is shown in Fig. 1a. In each subsequent computation, the convergent flowfield of the preceding computation was used as initial data. The variation of jet pressure imposed as a boundary condition on the nozzle exit corresponded to a $2^\circ$ change in the nozzle lip shock angle. The Mach reflection was preserved when increasing $p_j$ until the value $\alpha = 31^\circ$ was reached (see Fig. 1c and 1d). It is in good agreement with the theoretically predicted von Neumann angle $\alpha_N = 30.8^\circ$. In numerical simulation, an earlier transition to regular reflection can be expected because it is impossible to resolve a very small Mach stem whose height is comparable with the grid cell size.

After that, the pressure ratio was decreased. The reflection remained regular over the entire dual solution domain, and the transition to Mach reflection occurred when the angle of incidence was changed from $39^\circ$ to $41^\circ$ (Fig. 1f and 1a). It again agrees with the theoretical detachment angle. Thus, an evident hysteresis phenomenon was observed: within the dual solution domain, both regular and Mach reflections can be obtained depending on the initial conditions of the computation. This hysteresis is very similar to that observed earlier in numerical simulations of the flow around two symmetrical wedges and the flow in a converging channel.
Fig. 1. Computed flowfields (numerical schlieren images) for overexpanded jet flow at $M_j = 5$.

The computed flowfields show substantially different shock wave structures for regular and Mach reflections at the same value of $p_j/p_a$ compare Fig. 1b and 1f. In the case of regular reflection, the jet boundary is strongly curved, and intensive compression waves are focused at the centerline, causing the secondary increase in pressure, in addition to the primary peak in the reflection point of the nozzle lip shock. For Mach reflection, the only strong pressure rise is observed just behind the Mach stem. Further downstream, the pressure decreases as the flow accelerates again to supersonic velocities and later oscillates in a periodical system of compression and rarefaction waves within the jet core bounded by two slip surfaces emanating from the triple points.

Turbulent computations. A series of turbulent jet computations at $M_j = 5$ were carried out for a set of pressure ratios corresponding to incident shock wave angles $\alpha$ ranging from 31° to 42°, which covers the whole dual solution domain. The computation was started from the case where only regular reflection is possible ($\alpha = 31°$). After convergence, the pressure-ratio was progressively decreased step by step until a sudden transition to Mach reflection observed at $\alpha = 39.7°$. This angle is very close to the theoretical value of $\alpha = 39.3°$ obtained using the detachment criterion. The back transition was observed when changing the shock wave angle from $\alpha = 32°$ to $\alpha = 31°$. Note that for $\alpha = 32°$, we still have MR with a small but visible Mach stem whose nondimensional height is $s/h \approx 0.015$ (see Fig. 2). The angle $\alpha = 32°$ slightly exceeds the value of $\alpha = 30.8°$ theoretically predicted by the von Neumann criterion.
Fig. 2. Mach number contours of Mach reflection at $M_j = 5$, at $\alpha = 32^\circ$.

Fig. 3. Dependence of Mach stem size on shock wave angle.

Fig. 4. Turbulent kinetic energy flowfields for regular reflection at $\alpha = 31^\circ$ and Mach reflection at $\alpha = 42^\circ$ (right).

In general, the results of Euler and Navier – Stokes are very close to each other. Besides the angles of transition, the Mach stem heights $s$ in two simulations agree very well, see Fig. 3. The reason of the close agreement between inviscid and turbulent computations becomes clearer from Fig. 4 where the normalized turbulent kinetic energy $k/U^2$ flowfield is shown. For both regular and Mach reflection configurations, there is no turbulence on the jet axis (potential core). Turbulence production occurs essentially in the high shear region of the jet edge.

Finally, some words should be said concerning the possibility of experimental confirmation of the hysteresis phenomenon described above. As is known [4], the observation of the hysteresis for the two-wedge flow depends strongly on the level of disturbances in the wind tunnel used. A high level of disturbances can lead to an earlier transition to Mach reflection and even to a complete absence of the hysteresis phenomenon. It is difficult to predict a priori whether the overexpanded jet flow is a more or less noisy object and, consequently, more or less suitable for experiments on the hysteresis at the shock wave reflection transition. An additional problem can be that, at high Mach numbers, the nozzle lip shock is rather strong and can induce a boundary layer separation inside the nozzle; this separation should destroy the gasdynamic scheme considered in this paper.

**Conclusion**

The shock wave reflection transition in an overexpanded supersonic jet at the Mach number $M_j = 5$ has been numerically simulated. First, the Euler computations have been performed, which show that the transition from regular to Mach reflection and the back transition occur in agreement with the theoretical detachment and the von Neumann criteria, respectively. Thus, the dependence of the shock reflection type on initial conditions and the hysteresis phenomenon have been observed.
Further, numerical investigations of the transition between regular and Mach reflections in steady turbulent over expanded jets have been performed using the two-equation $k$-$\varepsilon$ model modified to account for compressibility effects. Very good agreement of the computational results with the theoretical criteria of transitions has been found as well as with the results of the inviscid simulations. Turbulence production has been concentrated within the jet boundary layer and did not affect the jet core flow substantially. As a result, the hysteresis effect has been also observed at increasing and decreasing the jet/ambient pressure ratio. The numerical study has revealed that, when the jet pressure ratio is sufficiently low, subsonic conditions appear downstream of the reflected shock for the regular reflection configuration. To better understand this phenomenon, which probably triggers the transition from regular to Mach reflection, further analytical and numerical studies for solving the problem of the interaction between the reflected shock and the jet boundary should be undertaken. Also, further investigations should consider the effect of the nozzle boundary layer and its possible separation on the shock wave configuration.

REFERENCES