GENERAL THEORY OF AXISYMMETRIC CONIC AND LOCALLY CONIC FLOWS AND REFLECTION OF STATIONARY SHOCK WAVES FROM THE AXIS OF SYMMETRY[1]

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The axisymmetric stationary conic and locally conic flows of ideal (not viscous and not heat-conducting gas) are considered. In polar coordinates $r\varphi$ with the center of conicity in an axis of symmetry the conical flows parameters are functions of only angle φ in infinite or finite parts of a physical plane. For locally conic flows this property is precisely realized in the center of conicity, and approximately – in its small vicinity. A necessary condition of local conicity is the distinction from zero of radial component value of a velocity vector when approach to the center of conicity at least in one of the rays φ = const. By virtue of it the uniform flow cannot be considered as a conic flow. The researched flows can be conically sub- and supersonic. In view of the remark on a uniform flow the change of their type is possible only over the shock wave (SW) with unique exception (joining of two conic flows of different types along C^+-characteristic). There are designed C^+-and C^--characteristics for locally conic flows which would arise when flow of a trailing part of a body rotation without deceleration in a sharpening point and when regular SW reflection from the axis of symmetry, and also for almost all conic flows. The consequences of the theory of locally conic flows are the proof of impossibility of SW regular reflection from an axis of symmetry and inapplicability for the proof of this fact of the reception, offered by A.A. Nikolsky. It is based on the integration of a compatibility condition along the C^--characteristic, which (under the assumption) would come to an axis of symmetry in a point of regular reflection. By the numerical integration of the Euler equations with using of crushed when approach to an axis of symmetry numerical grids there are determined the sizes of the Mach disks for initially weak SW. It is shown, that their sizes decrease faster, than under the square-law from initial SW intensity.

Introduction

The examples of axisymmetric conic flows (CF) are: the flow without angle of attack of a circular cone by the inert gas [1-7] and by the detonating mix [8], the flow in a diffuser of A. Busemann [1-7], A.A. Nikolsky's flow about the specially designed trailing part [3-7, 9] and that of G.L. Grodzovsky [3, 10] in limited removed from an axis of symmetry annular areas with one rectilinear border (SW section). For these flows in polar coordinates $r\varphi$ with the center in an axis of symmetry the parameters of a stream do not depend from r, by virtue of what the CF equations (CFE) are the ordinary differential equations with derivatives per φ. However when flow of already finite cone the conditions are possible, when for SW of weak family a flow near the cone surface is subsonic and derivatives per r are distinct from zero everywhere, including as much as small vicinity of an edge. Nevertheless the dependence of parameters from φ at the edge is still defined by the same CFE. Such locally conic flows (LCF) are strictly conic in the center of conicity and are close to them in its small vicinity. The same picture takes place for axisymmetric flow with attached SW of any sharpened bodies. In a small vicinity of the edge it is close, and in the edge is identical to a flow of a circular cone with the same angle of the sharpness.

In the book of Courant and Fridrichs [1] on the basis of CFE analysis in four phrases (section 157) the proof of the impossibility of stationary SW regular reflection from an axis of symmetry is planned: « Other problem which … contains conic flow, is *a reflection of conic SW*. The constant parallel flow is declined to an axis by the "coming" SW and again becomes parallel, passing through the "reflected" SW. It would be possible to construct such flow, supposing it to be purely conic between two SW and after reflected SW. Actually the flows of such type *do not exist*, that it is possible to see from the reasoning based on a sign of v_{uu} » (hereinafter u and v are axial and radial components of a gas velocity vector **V**, and $v_{uu} = d^2v/du^2$). Then authors [1] write: «Came out with the assumption, that all observed reflections … of SW are actually the Mach reflections, though the Mach disks are too small to be noticeable. It is confirmed to some extent with calculations of Ferri », and the reference to Ferri work which is the precursor of the report [11] is given. In it for annular channels with angles of rectilinear generatrix θ = – ε when ε = 5°, 9°, 13° and Mach number of the incident flow M_∞ = 1.6 the method of characteristics designs the supersonic flow behind the extended to an axis of symmetry SW, and the curve $y_{SW1}/y_1 = f(\varepsilon)$ is designed. Here y_1 and y_{SW1} are the radial coordinates of a forward edge of the annular channel and of a point, in which M = 1 behind SW, and the index «∞» marks parameters of the incident flow.

If we accept, that y_{SW1} is less than radius y_{DM} of a Mach disk (that is by the way absolutely unessential - see below), arising when irregular reflection, then results [11, 12] allow to assume reduction of value y_{DM}/y_1 when reduction ε and growth M_∞. For SW with appreciable initial intensity (for which the pressure ratio is more than 1.42) this assumption

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was confirmed with results of work [13] of 1974. In it the fields of supersonic flow were calculated with numerical integration of the Euler's equations, and the subsonic flow behind Mach disk was considered in one-dimensional approach. In [13] it is written: «It may be proved, that in axisymmetric flow the regular reflection of coming shock from an axis of a jet is impossible», and in connection with experiments [14] on SW reflection from an axis of symmetry it is told: « In … cases when in conditions of ideal gas the size of central shock (*a Mach disk*) should be small, owing to influence of viscosity the reflection … can occur without forming of a central shock».

The statement equivalent told in [1] about the impossibility of regular SW reflection from an axis of symmetry, is so briefly and practically in the same form resulted at the end of the devoted to axisymmetric conic flows § 16 of G.G. Cherny's book [4], that came out in 1988. However the proof of the impossibility of regular reflection with detailed CFE analysis and furthermore with their numerical solving was not resulted in it too. On the other hand, in 1949 A.A. Nikolsky [15] has offered extremely simple and elegant reception, which as it was treated till the latest time, instantly solved the given problem. Though in [15] the other problem was considered – the flow of peaked back edges of rotation bodies with angles of sharpness, distinct from zero, and the impossibility of their supersonic flow was proved, the transfer to problem of SW reflection from an axis of symmetry was seemed obvious. For the first time the reception of A.A. Nikolsky as a way of the proof of the impossibility of stationary SW regular reflection from an axis of symmetry is mentioned in work [16] of 1990. It is noticed in it, that the fact of the impossibility of stationary SW regular reflection from an axis of symmetry «is known, it can … be proved (*the reference to* [15]) by the integration of a compatibility condition along the characteristic coming to a point of the expected regular reflection». About the same proof from the beginning of 1970th the first author of the present report spoke in lectures for students of MFTI, what was reproduced in [6, 7].

Despite of the above-stated, the point of view is extended, that SW which initial intensity does not exceed some «threshold» size, are reflected from an axis of symmetry by a regular way. Such view of the given problem is confirmed by already mentioned experiments [14], either quite fresh results of the «through» calculation of flows with stationary SW reflection from an axis of symmetry [17].

Below first the general questions of the axisymmetric conic and locally conic flows theory are discussed, in particular those, which are obliged to realize when regular SW reflection from an axis of symmetry, and that were actually indicated by Courant and Fridrichs [1]. The results of of the wide set of locally conic and conic flows calculations are presented, including adequate to considered in [15] supersonic flow of a trailing part of a rotation body and being expected regular reflection of any families SW from an axis of symmetry. These calculations have shown, that application to them of A.A. Nikolsky's reception is incorrect and consequently does not prove the impossibility of such flows. The proof of it according to planned in [1] analysis is reduced to establishment of the absence of CFE solutions for which the stream flowing to an axis of symmetry along a trailing part of a rotation body or turned to it by the coming SW, could get again an axial direction in SW and in continuous LCF.

The CF calculations include the flow in diffuser of A. Busemann, the flow of a rotation body trailing part of A.A. Nikolsky and the extremely interesting CF of products of a gas mixture burning with the state equations of the perfect gas behind detonation wave of Chapman-Juget. For last G.G. Cherny and S.S. Kvashnina still in 1959[8] first have found out and have constructed automodelling decisions with the radiating detonation and shock waves, attached to a circular cone. It is remarkable, that such strong breaks of «one family» going from one point appeared are possible in normal gas (with positive «fundamental» derivative $\omega_{pp} \equiv (\partial^2\omega/\partial p^2)_s$, where ω is a specific volume, p is a pressure and s is a specific entropy).

In final section the data are resulted on the dependence of Mach disks sizes from the Mach number of the incoming flow M_{∞} and from the initial intensity of initially weak SW. These data received with explicit construction of coming SW and a Mach disk on grids, crushed to an axis of symmetry, show fast (more than square-law) reduction of a Mach disk radius when reduction of size ε. Discussed further questions are partially stated in recently published article [18].

1. The general theory and the results of locally conic and conic flows calculation

The described above properties of LCF are the consequences of the equations for pressure p and angle θ between the gas velocity vector **V** and the directed to an axis of symmetry the axis x of the cartesian coordinates xy. For axisymmetric not rotated flows of ideal gas in coordinates $r\varphi$ for any position of their origin (a point o) in an axis x they are representable in the form

$$[\mathrm{M}^2\cos^2(\varphi-\theta)-1]p_r = r^{-1}P(p_\varphi,...),\quad [\mathrm{M}^2\cos^2(\varphi-\theta)-1]\rho V^2\theta_r = r^{-1}\Theta(p_\varphi,...),$$
$$P(p_\varphi,...) = \mathrm{M}^2 p_\varphi \sin(\varphi-\theta)\cos(\varphi-\theta) - \rho V^2\theta_\varphi - \rho V^2\sin\theta\cos(\varphi-\theta)\sin^{-1}\varphi, \tag{1.1}$$
$$\Theta(p_\varphi,...) = (1-\mathrm{M}^2)p_\varphi + \rho V^2\theta_\varphi \mathrm{M}^2\sin(\varphi-\theta)\cos(\varphi-\theta) + \rho V^2\sin\theta\sin(\varphi-\theta)\sin^{-1}\varphi\,.$$

Here $V = |\mathbf{V}|$; $\rho = 1/\omega$ and a are the density and sonic velocity, p_r, p_φ, … are the partial derivatives per r and φ. Two more equations, representing the integrals of full enthalpy and entropy (issueing due to introduction of the flow function), are finite.

When arbitrary choice of a point o the right parts of the relations (1.1) are the values of the same order, as their left parts, and (1.1) are written down in coordinates $r\varphi$ the equations in partial derivatives. The other case, if o is the center of conicity. A necessary condition of local conicity is the difference from zero of y-component of velocity $v = V\sin\theta$ at least in one of the rays φ = const. For the bodies pointed in front or behind, θ is a semi-angle of a pointing distinct from zero, and for oblique SW, coming to an axis of symmetry (if such is possible) or leaving it (as when flow of the pointed bodies), θ is an angle behind SW. Due to this when $r \to 0$ in expressions for P and Θ there appear proportional $\rho V^2\sin\theta$ finite sums, which should compensate also finite p_φ and $\rho V^2\theta_\varphi$.. Therefore in a point o for all φ for which $V\sin\theta \neq 0$, equations $P = 0$ and $\Theta = 0$ should be true. Otherwise $|p|$ and $|\theta|$ when approach to a point o would grow proportionally $\ln|r|$. However p in stationary flows is finite because of the finiteness of entropy and full enthalpy. Therefore the equations $P = 0$ and $\Theta = 0$ or their consequences

$$p_\varphi = \frac{\rho V^2 \sin\theta\sin(\varphi-\theta)}{[\mathrm{M}^2\sin^2(\varphi-\theta)-1]\sin\varphi}, \quad \rho V^2\theta_\varphi = \frac{\rho V^2 \sin\theta\cos(\varphi-\theta)}{[\mathrm{M}^2\sin^2(\varphi-\theta)-1]\sin\varphi} \tag{1.2}$$

precisely describe LCF in the center of conicity and approximately in its small vicinity. In particular, so would be when regular reflection from axis of symmetry of any family oblique SW, and in a small vicinity of a point of reflection the coming SW and a number of other curves mentioned further would be close to rays φ = const.

The close reasoning, though without a mention of a necessary condition of local conicity (inequality $V\sin\theta \neq 0$) and with the equations (1.1) multiplied on $r/(\rho V^2)$, that in this case can disorient, and for M ≥ 1 and with the equations, equivalent (1.2), are resulted in [16]. However and up to [16] their consequences were considered obvious. So, when calculation of a flow of the pointed bodies of rotation with curvilinear generatrix the flow in a small vicinity of an edge with attached to it SW was replaced by conic even for angles of attack distinct from zero.

So, in the center of conicity for LCF or in a whole CF the equations (1.2) are valid or equivalent to them CFE in the known [1-7] form used further

$$v_{uu} \equiv \frac{d^2v}{du^2} = N\frac{1+v_u^2}{a^2v}, \quad v_u = -\mathrm{ctg}\varphi, \quad \varphi_u = \frac{N}{a^2v}, \quad N = a^2 - V_\mathrm{n}^2, \quad V_\mathrm{n}^2 = \frac{(u+vv_u)^2}{1+v_u^2} = (u\sin\varphi - v\cos\varphi)^2 . \tag{1.3}$$

Here all the parameters are functions of only x-component of velocity u, V_n is a projection of **V** to a normal to ray φ = = const, and for scales of velocity, density and pressure are taken the critical velocity and density a_* and ρ_* and $\rho_*a_*^2$. Gas is a perfect, for which $a^2 = [\gamma+1-(\gamma-1)V^2]/2$. As usual ratio of the specific thermal capacities is γ = 1.4.

The ratio $\mathrm{M_n} = V_\mathrm{n}/a$ has the same sense, that the Mach number determining the type of two-dimensional CF [6, 7]. On the assumption of this, the one-dimensional (axisymmetrical) CF with $\mathrm{M_n} > 1$ it is natural to name conically supersonic («CSupSF»), with $\mathrm{M_n} < 1$, for example, behind SW and on a cone - conically subsonic («CSubSF»), and a ray, along which $\mathrm{M_n}= 1$, - conically sonic. As apparently from the further, with the only exception, also described further, the continuous change of the sign N or, that is the same, of the type of axisymmetric CF is impossible as against of two-dimensional flows. According to told earlier, for validity of the given statement we shall not consider the uniform flow, parallel to an axis x, as CF (see below).

If we multiply the equations (1.1) by r, and then approach r to zero, we shall find, that equations $P = 0$ and $\Theta = 0$, their consequences - equality (1.2) and the equations, which are realized after transition in (1.3) to independent variable φ, are true also in those points of an axis x, in which $v \equiv V\sin\theta = 0$. All these equations, however, when absence of private derivatives per r, missed when $r = 0$ because of multiplication by r, give only that obvious information, that in such points of an axis x when $V \neq 0$ then p and θ do not depend on φ , and when $V = 0$, i.e. in a stagnation point - only p does not. When $V = 0$ the second equation (1.2), becoming identity, does not define the dependence θ on φ. Hence, for the description the flow in a vicinity of such points the transition to polar coordinates is useless. Whereas in variables xy the equations for p and θ in such points become

$$(\mathrm{M}^2-1)\frac{\partial p}{\partial x} + 2\rho V^2\frac{\partial\theta}{\partial y} = 0, \quad \frac{\partial p}{\partial y} = 0 .$$

In uniform (parallel to an axis x) flow an angle θ and all derivatives are identical zero. Therefore it satisfies also to the initial equations (1.1), and the equations of conic flow $P = 0$, $\Theta = 0$, and when independent variable φ also to all to their consequences. Thus, for any choice of a point o in an axis of symmetry there is $\mathrm{M_n} > 1$ in sector between C^-- and C^+-characteristics, coming to a point o, and $\mathrm{M_n} < 1$ outside of this sector with continuous transition through unit in its borders. The given case could be counted as an example of continuous change of «conic» flow type and of a sign on difference N, which is presented in the equations (1.3). Here, however, the change of $\mathrm{M_n}$ and N is connected only with orientation of a ray, to a normal to which a parallel to an axis x constant velocity of a flow is projected, and the flow in «the center of conicity» does not satisfy to the formulated above necessary condition of local conicity. By virtue of told it is senseless to count a uniform flow by CF.

The work with the first equation (1.3) and V_n^2 , defined on u, v and v_u , is usually considered preferable that without attraction of the second equation its integration allows to build the CF curves: $v = v(u)$ in a hodograph plane. However, actually, when equally simple numerical integration of one equation of the second order - the first equation (1.3) and of two the first order equations – the second and third equations (1.3) the use of two equations simplifies both the analysis, and construction of different CF.

When gas flows from left to right then for all problems of CF construction an angle φ decreases when y> 0. So, when flow of a cone it decreases from a value of SW inclination (φ_{SW}) up to semi-angle of a cone top ($\varphi = \theta_c$). If we mark parameters behind SW by an index «+» then behind the SW of finite intensity $v_+ > 0$, $V_{n+}^2 < a_+^2$ and $N_+ > 0$, i.e. the flow is conically subsonic and the right part of the third equation (1.3) is positive. Therefore for reduction φ the x-component of velocity u from its value u_+ behind SW is need to reduce up to beforehand unknown value u_c. Thus by virtue of the second equation (1.3) with a negative right part the y-component of the velocity v, and furthermore tgθ = $= v/u$ will grow. To take into account the isoentropy and isoenthalpy character of CF and the formula for a^2, it is possible to show, that

$$N_u = -N\frac{\sin 2(\varphi-\theta)}{v\sin^2\mu} - V\rho^3 a^4 \omega_{pp}\frac{\sin(\varphi-\theta)}{\sin\varphi}, \tag{1.4}$$

where μ is a Mach angle (sinμ = 1/M), and $\varphi - \theta \geq 0$ with equality being at generatrix of cone only. For perfect gas [6, 7]: $\rho^3 a^4 \omega_{pp} = \gamma + 1$, and $M^2 = 2V^2/[\gamma + 1 - (\gamma - 1)V^2]$. When $N > 0$ the right side of equation (1.4) is negative. Thus when cone flow N monotonically increases with decreasing of u, and the flow remains to be conically subsonic.

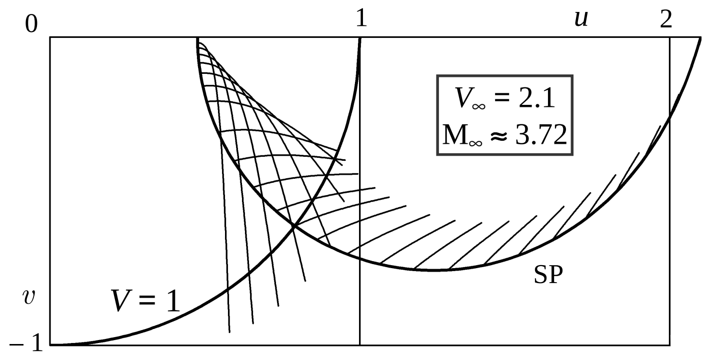

Fig. 1

The results of integration of the equations (1.3) for value of flow velocity before SW $V_\infty = 2.1$ ($M_\infty = 3.724$), are presented in Fig. 1. In it in a plane uv there are constructed the sonic circle $V = 1$, the bottom half of shock polar (SP), corresponding to going to an axis of symmetry SW of weak both strong families, and beginning in a polar CF curves. CF curves, beginning in SP when V> 1, describe CF [10] in finite not adjoining to an axis areas of a plane xy.

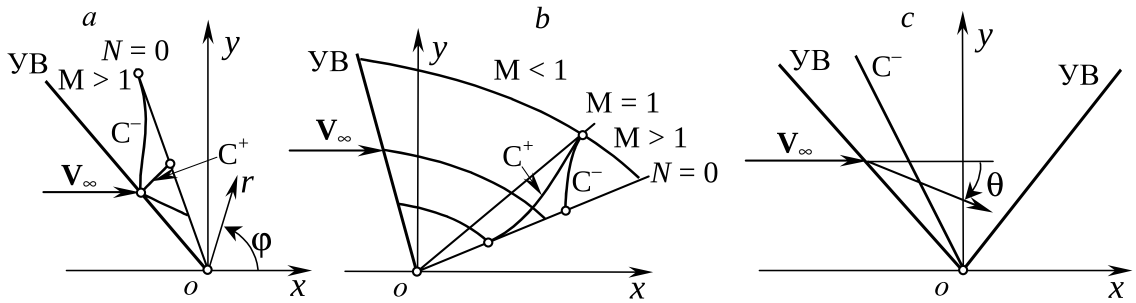

Fig. 2

According to the second equation (1.3) the direction of normal to CF curves in a plane uv (fig. 1, a) coincides with a direction of rays φ = const in a plane xy (fig. 2, a). The angles φ for points in a shock polar, i.e. angles of SW inclination in the designed cases are negative, behind SW $M_n < 1$, $N > 0$, $v_{uu} < 0$, CF curves are convex, and the normals to them turn clockwise when movement from points in a polar. So will be up to the ray $\varphi = \varphi_{N0}$, in which it will become zero the value $N = a^2 - V_n^2$.

Along the constructed in Fig. 1 CF curves v are negative, and the integration of the equations (1.3) is conducted up to size $u = u_{N0}$ and corresponding to it ray $\varphi = \varphi_{N0}$. As v simultaneously with N does not become zero, then by virtue of the third equation (1.3) it is impossible to continue the CF curve further. When preservation of a change u direction the angle φ begins to grow, and rays φ = const of plane *xy* will start to cover the CF sector, calculated up to it with new values of the flow parameters. If when achievement $N = 0$ to change *u* in the opposite direction then within errors of integration it will be repeated already calculated CF up to SW and corresponding to it the polar point. Though $V_n^2 = a^2$ when $\varphi = \varphi_{N0}$, this ray is not C+- or C--characteristic for along it none of compatibility conditions is true [6 , 7]

$$d\theta \pm \frac{\text{ctg}\mu}{\rho V^2} dp \pm \frac{\sin\theta \sin\mu}{y \sin(\theta \pm \mu)} dy = 0 \,. \qquad (1.5^{\pm})$$

Really, along such ray *p* and θ are constant, and when θ ≠ 0 the third sums are distinct from zero. Hereinafter when presence in the equations of two signs top (bottom) those correspond to C^+ (C^-)- characteristics.

For CF curves, beginning in a shock polar outside of the sonic circle in points, corresponding to SW of weak family, to SW ($\varphi = \varphi_-$) the sector of supersonic (M > 1), but conically subsonic flow (fig. 2, *a*) adjoins. When $\varphi = \varphi_{N0}$ in CF curve $M_n = 1$. For CF curves, beginning in shock polar inside a sonic circle (in the points, corresponding to SW of a strong family), to SW the sector of subsonic flow (fig. 2, *b*) adjoins. As however when $\varphi = \varphi_{N0}$ in CF curve $M_n = 1$, then along this ray generally $M > M_n = 1$, and when some $\varphi_{M1} \geq \varphi_{N0}$ there is a transition through a sonic velocity (M == 1).

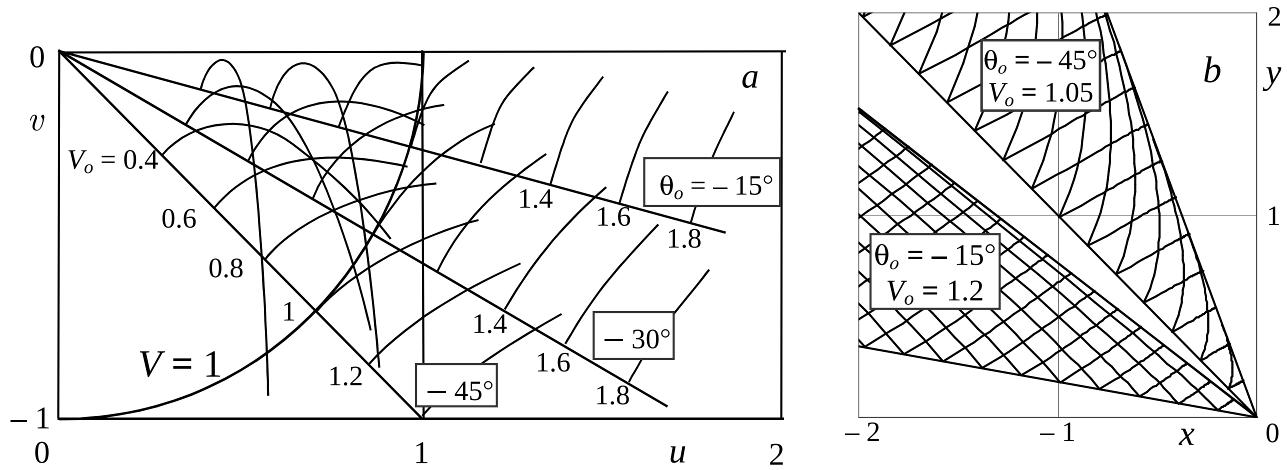

Fig. 3

If we admit a flow of a peaked trailing part of a rotation body with a nonzero angle of a sharpness ($\theta_o < 0$) without deceleration of the flow in its end point *o* such flow by virtue of inequality $V_o \sin\theta_o \neq 0$ is obliged to be locally conic for anyone $V_o > 0$. On a body by virtue of a non-permeability condition $N_o = a_o^2 > 0$, and continuous change of sign *N* is impossible. Told illustrates Fig. 3, *a* on which CF curves, designed for θ_o = – 15°, – 30°, – 45° for values V_o from 0.4 up to 1.8, are drawn. To surfaces of a body there correspond $\varphi_o = \pi + \theta_o$ and the initial points of CF curves, lying in straight lines $v/u = \text{tg}\theta_o$. Curves KT end when $N = 0$, that corresponds to $\varphi = \varphi_{N0}$. The designed CFL, remaining as well as on a surface of a body, conically subsonic, appreciably differ for super- and subsonic velocities V_o. When $V_o \geq 1$ the values φ_o and φ_{N0} are close, and the rays $\varphi = \varphi_{N0}$ being normal to CF curves, as well as the rays $\varphi = \varphi_o$, lay in the second quadrant of *xy* plane with the coordinates origin in a point *o*. When V_o <1 the direction of rays $\varphi = \varphi_{N0}$ comes nearer to a direction of an axis *x*. Nevertheless, all designed conically subsonic flows are not continued for its ray $\varphi = \varphi_{N0}$, and the flow of a peaked trailing part with $\theta_o < 0$ is possible only with full stagnation of a flow in a point *o*. In that case when finite change of angle θ in this point $V\sin\theta \to 0$ when $r \to 0$, and the equations (1.1) with the left and right parts of the same order remain to be the equations with partial derivatives in as much as small vicinity of the edge.

In a plane *xy* $C^{\pm}$-characteristics of CF are resulted by the integration of the equations [18]

$$\frac{y_u}{\sin(\theta \pm \mu)} = \frac{x_u}{\cos(\theta \pm \mu)} = \frac{y \sin(\varphi - \theta \pm \mu)}{V \sin\varphi \sin\theta \sin^2\mu} \,, \qquad (1.6)$$

when conclusion of which it was accounted, that along characteristics $dy/dx = \text{tg}(\theta \pm \mu)$. The results of integration for $\theta_o = -15^\circ$, $V_o = 1.2$ and $\theta_o = -45^\circ$, $V_o = 1.05$ are presented in Fig. 3, *b* with «grid» of $C^{\pm}$-characteristics between the rays $\varphi = \varphi_o$ and $\varphi = \varphi_{N0}$.

CF, realized in diffuser of A. Busemann [1-7] and when flow of a trailing part of A.A. Nikolsky [3-7, 9], adjoin to a uniform flow along C^-- and along C^+-characteristics with angles $\varphi = \varphi_\infty = \pi - \mu_\infty$ and μ_∞,, accordingly. In such characteristics $N = v = \theta = 0$, and in accordance with the equation (1.4) and the second equation (1.3), to the right of them

$(N_u)_{\infty+} = (V\rho^3 a^4 \omega_{pp})_\infty$, where the index "+" marks the right value of the discontinuous derivative. When a withdrawal from these characteristics, having taken for an independent variable a component of velocity v and having limited by the perfect gas, for definition N, φ and u we shall receive the equations

$$\frac{dN}{dv} = N\frac{\sin\varphi\sin 2(\varphi-\theta)}{v\sin^2\mu\cos\varphi} + (\gamma+1)V\frac{\sin(\varphi-\theta)}{\cos\varphi}, \quad \frac{d\varphi}{dv} = -N\frac{\mathrm{tg}\varphi}{a^2 v}, \quad \frac{du}{dv} = -\mathrm{tg}\varphi, \quad N = a^2 - (u\sin\varphi - v\cos\varphi)^2 . \qquad (1.7)$$

From here for definition N near to the discontinuous characteristic (when small v, N, $\delta\varphi = \varphi - \varphi_\infty$ and $\delta u = u - V_\infty$) we shall come to Ricatti equation (the top signs are for CF of A.A. Nikolsky, bottom signs - for CF of A. Busemann)

$$\frac{dN}{dv} = \frac{2\,\mathrm{m} f v}{v}N - \frac{c}{v}N^2 \pm b + gv + ... = \frac{2N \pm bv - cN^2\,\mathrm{m} f vN + gv^2}{v} + ...,$$
$$b = \frac{\gamma+1}{\mathrm{ctg}\mu_\infty}V_\infty, \quad c = \frac{2}{V_\infty^2\sin^2\mu_\infty}, \quad f = \frac{\gamma+1-4\cos^2\mu_\infty}{2V_\infty\sin\mu_\infty\cos^3\mu_\infty}, \quad g = (\gamma+1)\frac{\gamma+1-2\cos^2\mu_\infty}{2\cos^4\mu_\infty} \qquad (1.8)$$

with « + … » for small sizes of higher orders. When receiving of this equation the right part of the first equation (1.7) was transformed in view of that according to expression for N and to integral of full enthalpy in main orders δφ is the linear form, of v, N and δu, and by virtue of the third equation (1.7) $\delta u = -v\,\mathrm{tg}\varphi_\infty + ...$. A point $N = v = 0$ is a singular point of the first equation (1.7) and the equation (1.8).

In a vicinity of a singularity the solution of Riccati equation (1.8) is given by the decomposition

$$N = \mp\frac{(\gamma+1)V_\infty}{\mathrm{ctg}\mu_\infty}v + (\gamma+1)\frac{\gamma+1-2(2\gamma+3)\cos^2\mu_\infty}{2\cos^4\mu_\infty}v^2\ln|v| + Cv^2 + ... \qquad (1.9)$$

with a constant of integration C. The substitution of N from (1.9) to linearized per δφ the second equation (1.7) results to the linear equation for δφ, having integrated which we shall find

$$\delta\varphi = \frac{(\gamma+1)v}{V_\infty\cos^2\mu_\infty} \mp (\gamma+1)\frac{\gamma+1-2(2\gamma+3)\cos^2\mu_\infty}{4V_\infty^2\sin\mu_\infty\cos^5\mu_\infty}v^2\ln|v| \pm \frac{(\gamma+1)[5(\gamma+1)-2(4\gamma+1)\cos^2\mu_\infty] - 4C\cos^4\mu_\infty}{8V_\infty^2\sin\mu_\infty\cos^5\mu_\infty}v^2 + ... \qquad (1.10)$$

At last, after substitution (1.10) to the third equation (1.7) and the subsequent integration, we shall receive

$$\delta u = \mp\frac{v}{\mathrm{ctg}\mu_\infty} - \frac{(\gamma+1)v^2}{2V_\infty\cos^4\mu_\infty} \pm (\gamma+1)\frac{\gamma+1-2(2\gamma+3)\cos^2\mu_\infty}{12V_\infty^2\sin\mu_\infty\cos^7\mu_\infty}v^3\ln|v| \mp$$
$$\mp\frac{(\gamma+1)[41(\gamma+1)-14(4\gamma+3)\cos^2\mu_\infty] - 12C\cos^4\mu_\infty}{72V_\infty^2\sin\mu_\infty\cos^7\mu_\infty}v^3 + ... \qquad (1.11)$$

Solutions (1.9)-(1.11) with one constant of integration C allow design one-parametrical family of initial sites of corresponding CF, adjoining to a uniform flow. An angle φ for considered flows should decrease. Therefore δφ should be negative and in accordance with the main sum of solution (1.10) v also will be negative. Hence, for CF in diffuser of A. Busemann, which adjoins to a uniform flow along C^--characteristic, by virtue of the solution (1.11) with bottom sign the value of u, and with it also V will decrease (CF of deceleration). As, however, there is apparently from the solution (1.9) with bottom sign in this CF N becomes negative and there will be such, and a flow will be conically supersonic (CSupSF) up to ray $\varphi = \varphi_{N0} = 0$. For this ray the considered CF it is impossible to continue, but it is possible to direct SW along any beam $\varphi_\infty > \varphi > \varphi_{N0}$ and behind it N becomes positive, and the flow becomes conically subsonic (CSubSF). When and how it can be continued behind SW, it is discussed below. As opposed to this the CF of A.A. Nikolsky about the trailing parts with initial sites being described by solutions (1.9) and (1.11) with the top signs will be conically subsonic (CSubSF) flows of rarefaction up to the ray $\varphi = \varphi_{N0} = 0$.

Continuously adjoining to uniform flow CF of A. Busemann and A.A. Nikolsky are discussed in a number of monographs. However the examples of such CF, designed still in a precomputer epoch, till the latest time were only at these authors (one diffuser for $M_\infty = 3$ was in [2] and ten trailing parts for $M_\infty = 1.2$, 1.5, 1.7 and 2 were in [9]). Exceptions − two examples resulted in [18]. The extensive calculations executed for development of these examples, have found out the interesting features on which it is necessary to dwell.

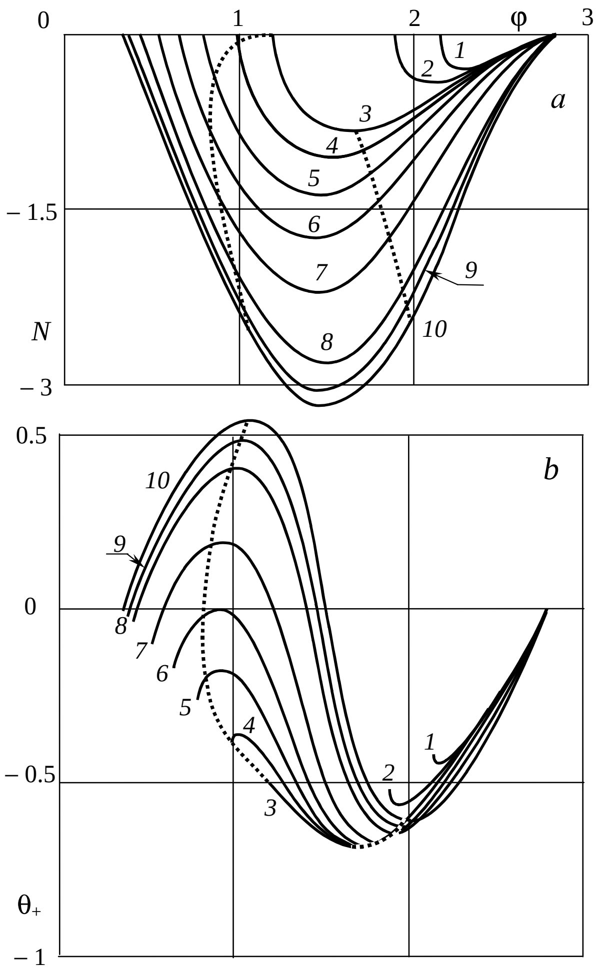

Fig. 4

Let's begin from the behavior of the integral curves of the first equation (1.7) and its consequences - the equations (1.8) in a vicinity of a node $N = v = 0$. The standard analysis with ignoring of square-law members in the numerator of the right part of the equation (1.8) results in the solution (1.9) without sum, proportional $v^2 \ln|v|$. The first solution, containing the logarithmic term, was constructed by A. Busemann [2] (see also [3, 18]). As $v \to 0$ this term prevails above Cv^2 with a constant *C*, determining one-parametrical family of solutions. The account of logarithmic term in the solution (1.9) changes the expressions (1.10) and (1.11) for φ and *u* and result in the third derivatives per *u* from v, *N* and φ in the initial characteristic become infinite (when $u \to V_\infty$). Because of it also the correct definition of the influence on the solution of size *C* is possible only when appreciable removal from the node, i.e. when not so small $\Delta = |v|$. For check of decomposition (1.9)-(1.11) accuracy when chosen initial $v = -\Delta$ under the formulas (1.10) and (1.11) there were defined φ(–Δ) and *u*(–Δ), and then with these initial values the equations (1.3) were integrated per *u* up to $u = V_\infty$ (or up to $v = 0$). In typical examples ($M_\infty = 3$, $-50 \le C \le 50$, $\Delta = 0.01$) the difference from zero of *N* and v when $u = V_\infty$ (or δφ and δ*u* when $v = 0$) did not exceed 10^{-6}. After so exact for given V_∞ «going to node» the equations (1.3) with the same initial φ(–Δ) and *u*(–Δ) and with different *C* were integrated sideways decreasing *u*. Here, as well as in [2], $\gamma = 1.405$.

The advanced approach allows find with high accuracy the dependence of the solution from M_∞ and constant *C*. For $M_\infty = 3$ and different *C* the results are submitted in Figs. 4-7. Continuous curves in Fig. 4 are designed for $-C =$ = 31.9731 (*1*), 31.9781 (*2*), 31.9831 (*3*), 31.9860 (*4*), 31.9931 (*5*), 32.0131 (*6*), 32.1130 (*7*), 33.6131 (*8*), 40 (*9*) and 100 (*10*). In Fig. 4, *a* they give the change *N* from $N = 0$ when $\varphi = \varphi_\infty \approx 2.802$ up to $N = 0$ when $\varphi = \varphi_{N0}$. Continuous curves in Fig. 4, *b* for the same *C* would show angles of an inclination of a velocity vector θ_+ for SW, directed along corresponding rays φ = const. For curves *1-5* these angles are negative, in a curve *6* where $C = -32.0131$, for the first time in one point θ_+ becomes zero, and when $C < -32.0131$, as for curves *7-10*, there are two such points with positive angles θ_+ between them. Dotted curves in Fig. 4 connect the points, corresponding to sonic velocity of the flow behind SW ($M_+ = 1$). Between them SW are the waves of strong family with $M_+ < 1$. From $\varphi = \varphi_\infty$ up to the right dotted curve and from the left dotted curve up to $\varphi = \varphi_{N0}$ there are realized SW of weak family with $M_+ > 1$. It is visible, that the qualitative change of the solution takes place in the narrowest interval of values of a constant *C*. Out of it such changes are absent. The curves for $C > -31.9731$, are pulled together to point $\varphi = \varphi_\infty$, essentially not differing from curves *1* and *2*. For $C < -33.6131$, the growth of |C | results in fast approach of the results as it is visible from comparison of curves *8*, *9* and *10*.

The traditional diffuser of Busemann (TDB) with a uniform flow behind SW, going from the coinciding with the coordinates origin the center of conicity, comes out when $\theta_+ = 0$. According to Fig. 4, *b* for $M_\infty = 3$ such diffusers it would be possible to design for $C \le -31.9731$ (to equality there corresponds the unique TDB with a subsonic flow at the exit). For each smaller value of *C* there is the pair of TDB, and already for small reduction of *C* - one is with SW of the strong ($\varphi = \varphi_{SW1}$), and another is with SW of the weak ($\varphi = \varphi_{SW2}$) families. Before the first SW, i.e. when $\varphi_{SW1} \le \varphi \le \varphi_\infty$ the both solutions coincide. In Fig. 5, *a* the example of such CF, corresponding to curves *7* in Fig. 4, is resulted ($M_\infty = 3$, $C = -32.1130$). When absence of marking the axes *x* and *y* because of the flow conicity in Fig 5, *a* there are shown flow lines, C^+- and C^--characteristics, including initial one («∞»), the ray $\varphi = \varphi_{N0}$ («*0*») and SW of strong («*1*») and weak («*2*») families, behind which $\theta_+ = 0$. In approach of an ideal gas it is possible to replace any flow line before SW *1* or *2* by a contour, continued behind SW by a horizontal straight line. In Fig. 5, *a* one such flow line is drawn by a thick curve, and its two continuations - by the dashed straight lines *1* and *2*. Turned out in this example TDB with $\varphi_{SW1} \approx 70°$ ($M_{-1} \approx 2.09$) and $\varphi_{SW2} \approx 34°$ ($M_{-2} \approx 1.80$) when constriction per $y \approx 0.64$ (*1*) and 0.53 (*2*) retard the flow up to $M_{+1} \approx 0.60$ and $M_{+2} \approx 1.48$. In Fig. 5 and further the scales by axes *x* and *y* are identical, and the directions of change *u*, resulting to reduction of polar angle φ, are shown by arrows (↑ is growth, ↓ is reduction).

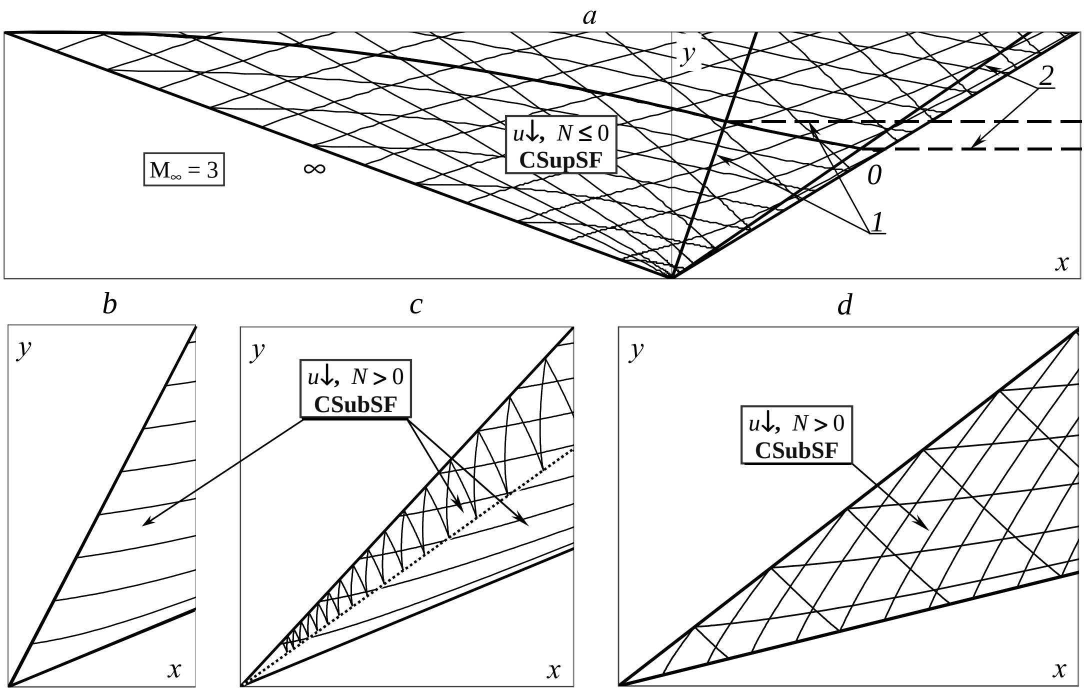

Fig. 5

To each SW with $\varphi_{SW2} < \varphi_{SW} < \varphi_{SW1}$ behind which according to Fig. 4, *b* $\theta_+ > 0$, it is possible to attach CF, realized when flow of a circular cone with top in the same center of conicity. Such CF turns out by integration of the equations (1.3) from $\varphi = \varphi_{SW}$, $u = u_+$ and other parameters for SW in a direction of reduction *u* up to realization of equality $\varphi = \theta$. The resulting diffusers unify A.Busemann's both solutions [2].

For the same $M_\infty = 3$ and $C = -32.1130$, the results of construction of three such CF are presented in Figs. 5, *b-d* for $\varphi_{SW} \approx 63°$ (*b*), 48° (*c*) and 38° (*d*). There are drawn SW (the left straight lines), the cones generatrixes (the right straight lines), flow lines, and when $M \ge 1$ – C^+- and C^--characteristics. Before each SW the flow lines and the $C^\pm$-characteristic are identical to represented in Fig. 5, *a*. To the left of SW a flow is conically supersonic (CSupSF), on the right it is conically subsonic (CSubSF). In Fig. 5, *b* the cone with $\theta_c = 23°$ is flowed round with SW of the strong family ($M_+ \approx 0.71$, $M_c \approx 0.63$). In Fig. 5, *c* the cone with practically same θ_c is flowed round with SW of weak family. However a flow, that is supersonic behind SW ($M_+ \approx 1.05$), being retard (a dashed straight line is the sonic line M = 1), becomes subsonic ($M_c \approx 0.97$). At last, in Fig. 5, *d* the cone with $\theta_c \approx 14°$ is flowed round by a fully supersonic flow ($M_+ \approx 1.33$, $M_c \approx 1.26$).

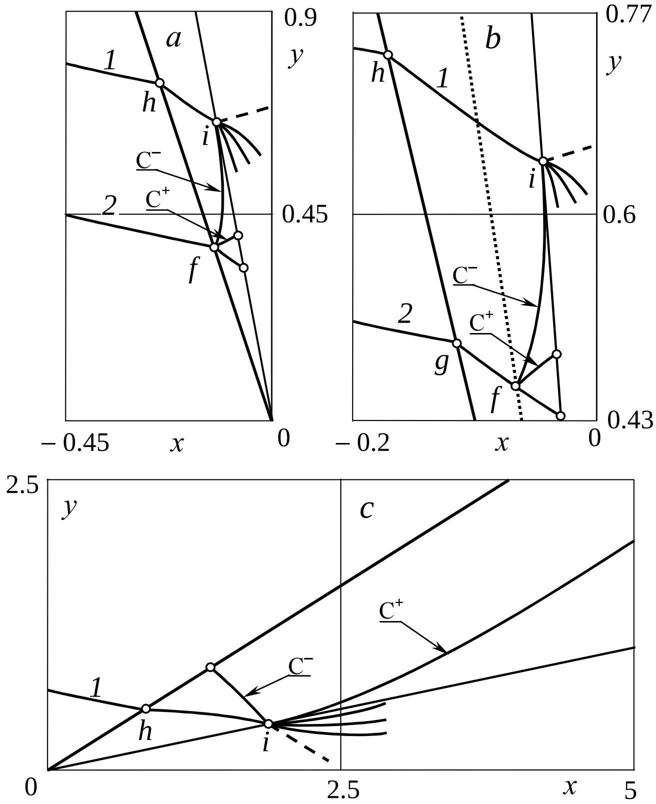

Fig. 6

For realization (in approach of ideal gas) of CF in TDB (fig. 5, *a*) and in diffusers with a cone (figs. 5, *b*-*d*) alongside with the setting of their contours along the flow lines it is necessary to provide the certain conditions of the flow at the exit of diffuser. It is simple, if the flow at the exit is supersonic (uniform, as in TDB with SW of weak family, or non-uniform, as in Fig. 5, *d*, where with *x* growth the non-uniformity of a flow at the exit quickly decreases). Rather more difficultly there is in case of Fig. 5, *c* and it is the most difficultly for CF with SW of strong family. Theoretically for realization of such CF the pressure at the exit should coincide with the designed one.

The flows, represented in Fig. 5, do not exhaust the all variety of CF, connected with diffuser of Busemann. In Figs. 6 and 7 the other CF are submitted, constructed in the annular channels, formed by the flow lines of this flow and CF for SW, such, that behind them according to Fig. 4, *b* an angle θ_+ is negative. In Fig 6 for $M_\infty = 3$ and $C = -32.1130$ the results of calculation of rarefaction CF behind SW, directed along the rays $\varphi_{SW} \approx 108°$ (*a*), 103° (*b*) и 33° (*c*) are presented. When equal scales for axes *x* and *y* there are drawn SW (the left straight line), the ray $\varphi = \varphi_{N0}$ (the right straight line) in which $N = 0$, two by two (*a* and *b*) or by one *(c)* flow lines (curves *1* and *2*) and by one C$^+$- and C$^-$-characteristic of CF. A curve *1* is the flow line, which has gone from a point $y = 1$, $x = -\mathrm{ctg}\mu_\infty = -(M_\infty^2 - 1)^{1/2} \approx -2.83$ in the boundary C$^-$-characteristic («∞» in Fig. 5, *a)*. For SW of weak or strong families the values of M_+ are: ≈ 1.05 (*a*), 0.93 (*b*) и 1.54 (*c*). In all cases the CF when $\varphi_{N0} < \varphi \le \varphi_{SW}$, remaining conically subsonic, are accelerated (in a case *b* – up to M > 1 with transition through a sonic line, shown by the dashed straight line in Fig 6, *б*).

In Fig. 6, *a* the considered CF is realized in the annular channel, limited (in a plane *xy*) from above by a flow line *1*, at the left – by the boundary C$^-$-characteristic, and on the right by C$^-$-characteristic *if*. The last one begins in a point *i* of crossing of the flow line *1* and the ray $\varphi = \varphi_{N0}$ and comes to an end in a point *f* in SW. To a point *f* there comes also a flow line *2,* which limits this CF from below. For its realization in a point *i* of a contour of the top wall of the channel the turn is made, with formation of a fan of rarefaction waves. Walls after a turn are drawn by strokes, and a fan of rarefaction waves – by the curves with the center in a point *i*. The CF in a triangle *ifh* – is the generalization of CF, constructed in [10], and the analogue of unreliazable LCF, represented in Fig. 2, *a*. The difference of CF in Fig. 6, *b* that as bidimentional (axisymmetrical) it is mixed with M < 1 between SW and rectilinear (but not straight, i.e. not normal to axis *x* and to a velocity vector) sonic line and with M > 1 from a sonic line up to C$^-$-characteristic *if*. As against of the mixed axisymmetrical flow, which can be observed when flow of a circular cone (fig. 5, *c*), here the subsonic flow precedes to the supersonic one.

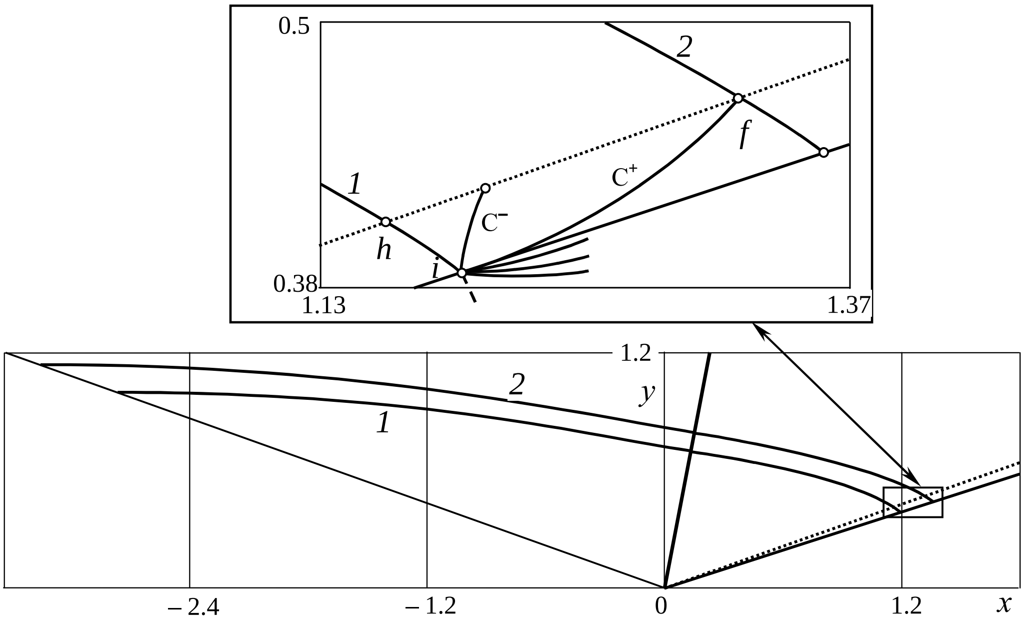

Fig. 7

If in CF of rarefaction behind SW $\varphi_{N0} < \pi/2$, then *if* – is a piece of the C^+-characteristic, instead of C^--characteristic, as in Figs. 6, *a* and *b*. Thus a flow line *2* (and the point *f*) is located not under, but above a flow line *1*. Such CF are resulted in Fig. 6, *c* ($M_+ \approx 1.54$, $\theta_+ \approx -0.044$) with the flow line *2*, laying outside of figure ($x_f \approx 15.4$, $y_f \approx 9.83$ are the coordinates of a point of arrival to SW of the C^+-characteristic *if*), and in Fig. 7. Figure 6, *c* is designed for the point of the curve *7* in Fig. 4, close to its left end. It shows the interesting feature of such CF: the closer φ_{SW} to φ_{N0}, the remoter from a flow line *1* is the flow line *2* and the greater turns out a triangle *ifh*. Actually, however, because of the decelerating of the supersonic flow, which is flowing round a "dashed" rotation body, will arise SW which, coming in the mentioned triangle, will limit the constructed CF on the right.

In Fig. 7, designed for $M_\infty = 3$, but $C = -32.2381$, the CF behind SW ($\varphi_{SW} \approx 79°$), except for narrow sector ($19.45° \geq \varphi \geq \varphi_{N0} = 17.99°$), is subsonic, as in Fig. 2, *b* and 6, *c*, it is limited on the right not by C^-, but C^+-characteristic. It is limited from above by the flow line *2*, as well as CF in Fig. 6, *c*. The angle of the flow inclination changes over SW from $\theta_- = -10.4°$ up to $\theta_+ = -9.5°$, and then turns clockwise almost up to $-38°$ in the ray $\varphi = \varphi_{N0}$.

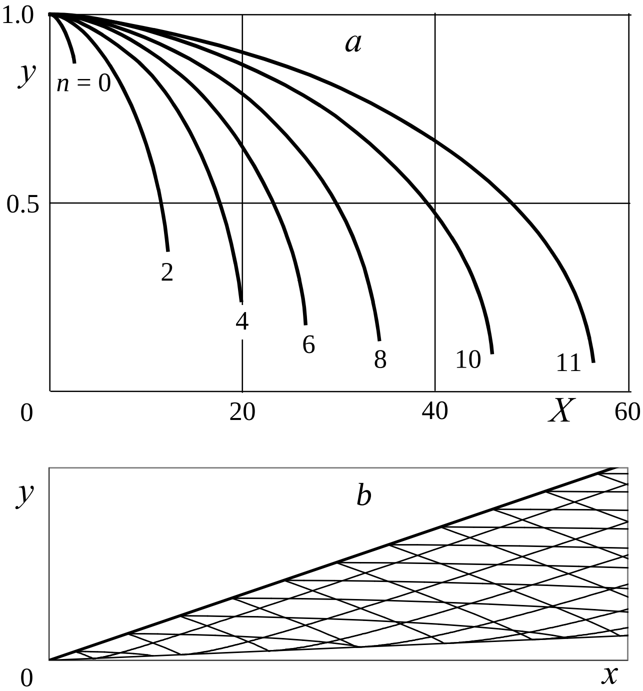

Fig. 8

Conically subsonic CF of rarefaction [9] with $\theta \le 0$ and $M \ge M_\infty$ (equalities are in the initial characteristic) adjoin to undisturbed flow along the C^+-characteristic. Their construction is carried out with attraction of decomposition (1.10) and (1.11) with the top signs and with $\delta\varphi = \varphi - \mu_\infty$. As the calculations have shown, here as against of CF [2], the solutions representing the interest for profiling of the trailing parts, are resulted for positive values of constant C. Thus when its monotonic growth the continuous well visible change of CF is observed. Told illustrate the flow lines in Fig. 8, *a*, designed for $M_\infty = 3$ and values $C = 200n^2$ when n = 0-11. When identical scaling of coordinates x and y the flow lines go out of point $x = \text{ctg}\mu_\infty \approx 2.83$, $y = 1$ of initial C^+-characteristic, and $X = x - \text{ctg}\mu_\infty$. Here and everywhere further γ = 1.4.

In Fig. 8, for $M_\infty = 3$ and $C = 300$ there are drawn the flow lines, C^+- and C^--characteristics and the ray $\varphi = \varphi_{N0}$ = = 2.46°, from which go out, concerning it, all the C^+-characteristics of this CF. In the specified ray M = 3.530, θ = = – 14°, and the size y of flow lines is equal to 0.5 of its value in initial C^+-characteristic. All the C^+-characteristics, going out from the ray $\varphi = \varphi_{N0}$, with removal from the origin of coordinates cross any beam with $\varphi < \varphi_\infty$.

The last example is the CF behind the detonation wave of Chapmen – Guget (DW_J) when flow of a circular cone [8]. As it was already marked, when flow of a cone with a finite angle at the top by the inert gas the flow behind SW up to a surface of cone is conically subsonic and $N > 0$. Other situation is when flow of a cone by the burning mixture with DW_J, for which $v_J > 0$, but $N = 0$. Behind DW_J the right part of the equation (1.4) is still negative, however N when removal from DW becomes positive for reduction u and negative when its growth. It is important, that according to the third equation (1.3) in both cases φ will decrease, and, hence, for DW_J the integration of the equations (1.3) is possible in the both direction: with reduction and with growth u. In the first case, according to the second equation (1.3), v grows, $\text{tg}\theta = v/u > 0$ also grows and the concave flow lines when $u \to u_c$ come nearer to a ray $\varphi = \theta_c$, coinciding with the generatrix of a cone. In it by virtue of a nonpermeability condition $V_n = 0$ and $N = a^2 > 0$. In such solution between DW_J and a surface of cone N is positive, and the flow is conically subsonic. Though in flow behind DW_J the direction of ray $\varphi = \varphi_J$ coincides with a direction of the C^+-characteristic, along it the compatibility condition (1.5^+) is not satisfied. Therefore, as well as the sonic lines considered earlier, the ray $\varphi = \varphi_J$ is the envelope of C^+-characteristics, going to it from below by a tangent.

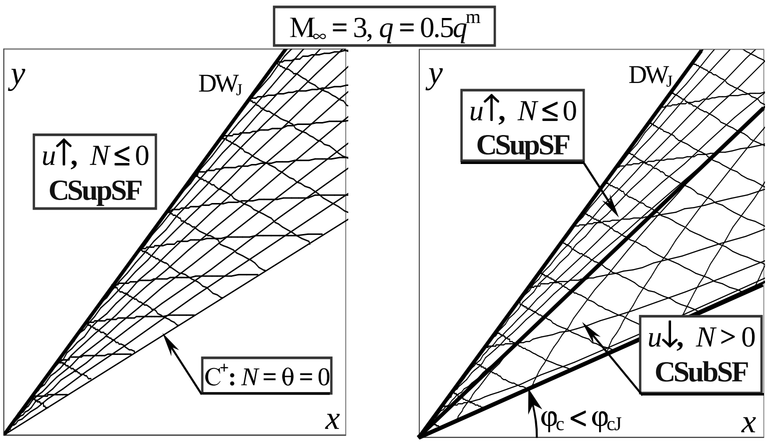

Fig. 9

When integration of the equations (1.3) in a direction of growth u according to the second equation (1.3) v decreases, $\text{tg}\theta = v/u > 0$ also decreases and now the convex flow lines at the some $u = V_0 > u_J$ and $\varphi = \varphi_0 < \varphi_J$ can become horizontal: $v = \theta = 0$. On the other hand, the sign of the right part of the equation (1.4) when N which, becoming negative, at first grows by the module, will inevitably change because of reduction v, and together with it, of the first sum. For this reason, since the some u, the right part of (1.4) will become positive, and the negative N will start to grow (decreasing by the module). Because of the decreasing v in a denominator of the first sum in the right part of (1.4) the growth N will be with acceleration.

It is natural to consider an opportunity of the simultaneous equal to zero v and N when $u = V_0$. In this case $\varphi = \mu_0$, where μ_0 is a Mach angle for $u = V_0$. Due to the fact, that on such ray parameters are constant and a component of velocity v is equal to zero, along it the compatibility condition (1.5^+) for C^+-characteristics is satisfied. Hence, the ray $\varphi = \mu_0$ is the C^+-characteristic, and not the envelope of such characteristics, as also having a characteristic direction, but not satisfying to the compatibility condition (because of $v > 0$) ray $\varphi = \varphi_J$. The executed above analysis of the first equation (1.7) and the solution (1.9) have shown, that the singular point $N = v = 0$ is the node. The node, collecting the integral curves of the first equation (1.6), provides when numerical construction of considered CF the simultaneous satisfying of equalities $N = 0$ and $v = 0$. In this case, however, as against the considered above examples, the values of V_0 and μ_0, replacing in the equation (1.8) and solutions (1.9) - (1.11) those of V_∞ and μ_∞, are beforehand unknown and are found in the process of getting solution.

When flow of a cone by the supersonic flow of a burning mixture the Ch-J detonation is possible not only when strictly certain semi-angle of a cone («Ch-J angle» – φ_{cJ}), but also for smaller angles. In the second case to DW_J there adjoins conically supersonic rarefaction flow limited by the conic SW, and to it adjoins up to the surface of a cone - conically supersonic compression flow. As it was already marked, such CF is the first example of automodelling flow with two strong breaks (DW_J and SW) of «one family», diverged from one point. If q is the heat of reaction, attributed to a square of velocity of the incoming flow, the stationary DW_J is possible when $q = kq^m$, where $k < 1$ and

$$q^m = (1 - M_\infty^{-2})^2 / [2(\gamma^2 - 1)].$$

For $M_\infty = 3$ and $k = 0.5$ and corresponding to them DW_J in Fig. 9 the results are submitted, which are turning out, if the equations (1.3), (1.6) and similar to last the equations of flow lines

$$x_u = \frac{-yN\cos\theta}{a^2 \upsilon \sin\varphi \sin(\varphi - \theta)}, \quad y_u = \frac{-yN\sin\theta}{a^2 \upsilon \sin\varphi \sin(\varphi - \theta)} \tag{1.12}$$

to integrate from $u = u_J$ for DW_J aside of the growth u. When conclusion of the equations (1.12) it was taken into account, that along the flow lines $dy/dx = \mathrm{tg}\theta$. In a plane xy there are drawn DW_J, C^+-and C^--characteristics and the flow lines. The scales in axes x and y are identical and the numbers, distinct from zero, in axes are absent (under the conicity of the flow). The left Fig. 9 corresponds to full acceleration of conically supersonic flow up to C^+-characteristic ($\varphi = \mu_0 \approx 29°$), in which $N = \theta = 0$ and $\mu = \mu_0$. To the right of it there is a uniform axial flow with the same $\theta = 0$ and μ. The flow lines of a constructed CF are the convex curves, and the C^+-characteristics, concerning DW_J in the initial points, leave it upwards and to the right: from DW there are propagated the rarefaction waves, drift by a supersonic flow away from DW_J.

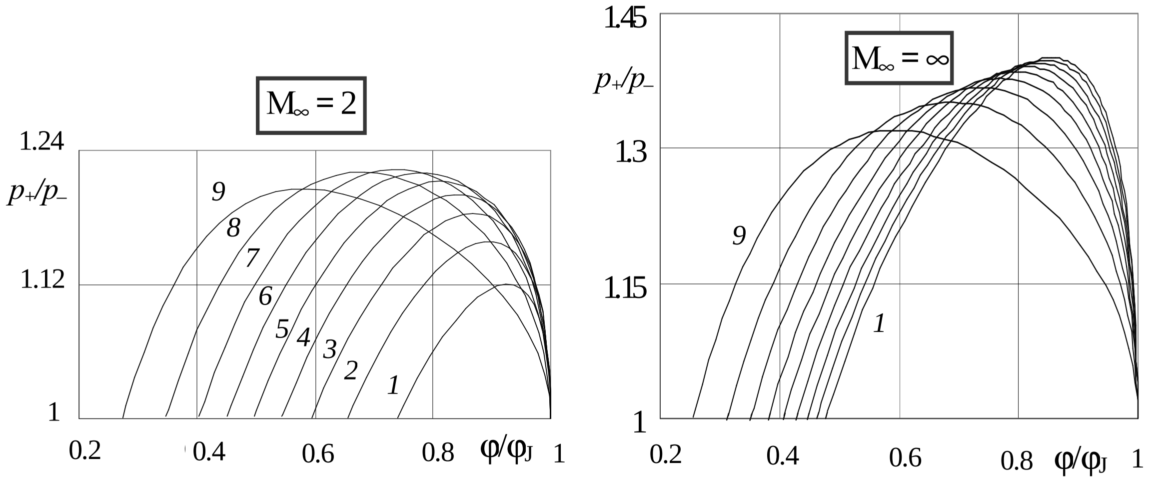

Fig. 10

As against of the left Fig. 9, corresponding to a flow of the «cone» with a zero angle at the top, in the right Fig. 9 there are submitted the results of a flow of a cone calculation with semi-angle at the top $\theta \approx 22°$, smaller than $\varphi_{cJ} \approx 39°$. In this case the represented in the left Fig. 9 conically supersonic rarefaction flow is kept up to SW with the angle $\varphi \approx$ $\approx 40°$. By value f it and the known parameters of the rarefaction CF before SW the parameters behind SW are defined, including the x-component of the velocity u_+. By the integration of the equations (1.3), (1.6) and (1.12) from $u = u_+$ aside of reduction u till satisfying of the equality $\varphi = \theta \approx 22°$ there is constructed a conically subsonic compression flow between SW and the cone generatrix. The flow lines of this CF are the concave curves: over the SW the curvature of flow lines changes a sign.

The intensity of SW – the ratio of pressure p_+/p_- depends on the size of M_n so, that $p_+/p_- = 1$ when $M_n = 1$. The SW of finite intensity (with $p_+/p_- > 1$) behind DW_J are possible when $0 < k < 1$ and $\mu_0 < \varphi < \varphi_J$. For $\gamma = 1.4$, $0 < k < 1$ and two values of Mach number $M_\infty = 2$ and ∞ the dependence of p_+/p_- from $\mu_0/\varphi_J \le \varphi/\varphi_J \le 1$ is shown in Fig. 10. The curves *1*, *2*, …, *9* in it correspond to values $k = 0.1, 0.2, \ldots, 0.9$. When fixed k and φ/φ_J the intensity of SW is monotonically growing function of M_∞. However even when $M_\infty = \infty$ for $k = 0.1$ the maximum of the ratio $p_+/p_- = 1.401$. With reduction γ it slightly decreases (up to 1.373 and 1.357 for $\gamma = 1.2$ and 1.1).

Let's dwell on the mentioned above exclusive situation with continuous change of sign N and the type of CF. Such situation is realized, if to constructed above CSupSF along the C^+-characteristic – the ray $\varphi = \mu_0$ to attach the CF of

A.A. Nikolsky [9]. In this CF u continues to grow, and a component v, becoming by virtue of the second equation (1.3) negative, also grows by the module. In the result the velocity of gas increases, pressure and density fall, the Mach number grows, but, despite of this, the attached CF, being the rarefaction flow appears to be conically subsonic.

From below of the flow the CF of A.A. Nikolsky is limited to the «sonic» ray $\varphi = \varphi_{N0} < \mu_0$, which, as well as the DW_J, having a characteristic direction ($N = 0$), because of $v \neq 0$ is not the C^+-characteristic. Each C^+-characteristic going out from it by the tangent in an initial point to the left and upwards, when big enough y crosses any ray $\varphi < \mu_0$.

In the considered ideal statement the described combination of two CF corresponds to a flow of the going to the left semi-infinite cylinder continued after crossing with DW_J by a flow line of a full (up to $v = 0$) rarefaction flow [8], and then by the flow line of CF [9]. The same curve constructed from two finite pieces of smoothly joined flow lines can be considered as an external contour of a body with a channel, flowed by a detonating mixture.

2. Reflection of steady shock waves from axis of symmetry

The mentioned above reception of A.A. Nikolsky [15] rests on a compatibility condition (1.5^-), holding true along C^--characteristics. If SW is reflected from an axis of symmetry on a regular basis (fig. 2, *c*), then behind it $\theta \neq 0$, and the factor before dy/y in the equation (1.5^-) when approach to axis of symmetry is remained the sign-determined function. Having integrated the equation (1.5^-) along a section of coming to an axis x C^--characteristic, we shall receive unlimited integral from the sum with dy/y which cannot be compensated by the finite integrals from two first sums. On the basis of it the conclusion is deduced about irregular reflection with formation of a Mach disk. First quite like this in [15] there was proved the impossibility of a supersonic flow of axisymmetric trailing parts with a finite angle of a point.

So simple, elegant and, at first sight, strict proofs, actually rest on an assumption about the existence of finite sections of C^--characteristics coming to the corresponding points of an axis of symmetry. However, as shown above, in the vicinities of similar points would inevitably be realized the LCF, in which such sections are not present. Really, in LCF in Fig. 2, *a* and 3, *b* or in their angular sector with $M > 1$ in Fig. 2, *b* there absent C^--characteristics of finite length coming to an axis of symmetry as it is drawn in Fig. 2, *c*. The lengths of sections of C^--characteristics crossing the supersonic (by M) sectors, when approach to axis of symmetry, tend to zero, and the ratios of ordinates of their end points - to finite constants. Thus, as against Fig. 2, *c* when the situations represented in figs. 2, *a* and *b* and 3, *b* the integration of a condition (1.5^-) along such sections does not result in any contradictions. The solutions with flow of the trailing part without full stagnation of a stream and with regular reflection of any family SW does not exist because of the impossibility of their continuation up to conically supersonic flow with $V_n > a$ and up to reflected SW which would turn a stream along an axis x. Just about this in connection with SW reflection though without detailed consideration it is written in monographs of Courant and Fridrichs [1] and G.G. Chernyi [4].

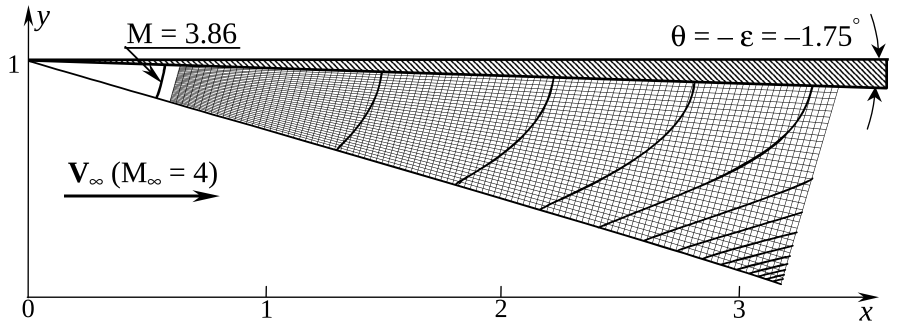

Fig. 11

The definition of the Mach disks sizes was carried out by numerical integration of the Euler equations for axisymmetric flow of ideal gas. The stationary solutions were executed by the pseudo-steadying per time (because of the different steps of integration for cells of the different sizes) with use of finite-difference scheme of S.K. Godunov [19] modified with account of reasons [20-22] and with explicit construction of coming SW. Before its reflection from an axis x there are no strong discontinuities in considered flow, therefore the nonmonotonic variant of the scheme was applied at first. In subregions of influence of the smeared reflected SW and tangential discontinuity at the final stage of the steadying it was turned on the monotonizator [20], and because of possible occurrence of symmetry of a Mach disk near the axis the algorithm close to described in [23] was applied to SW construction. It was used a multiblock grid with one block such as represented in Fig. 11 and several blocks with much finer cells in the field of SW reflection. As a result a Mach disk contained about 50 cell nodes. The right border of the calculation field was choused so, that the x-component of the velocity was supersonic in it.

Calculations were carried out for $M_\infty = 2$, 2.5, 3 and 4 and $\varepsilon = 1.75^\circ$ and 3.5°. In Fig. 11 for $M_\infty = 4$ and $\varepsilon = 1.75^\circ$ there are presented the most rough computation grid, SW and over the step $\Delta M = -0.02$ – isomachs of the flow outside of the vicinity of a reflection point and region of its influence. The longitudinal net lines converge to an edge of a ring. Near the edge the net lines are not shown in view of their merge.

The results processing of numerical integration of the Euler's equations and of ordinary differential equations of nonlinear theory, developed for weak SW in [18], has shown, that for an estimation of SW amplification the dependence is true

$$\vartheta \equiv -\theta/\varepsilon \approx 1/y^{1/n}, \tag{2.1}$$

similar to the formula of linear theory, but with $n > 2$. In the designed variants 2.2 < n <2.5.

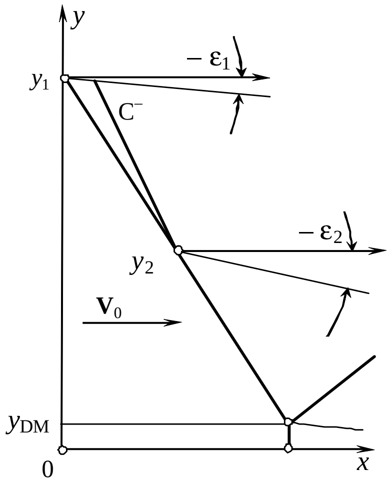

Fig. 12

The formula (2.1) allows when fixed M_0 and γ to find the dependence of a Mach disk size from value of ε, i.e. from initial intensity of weak SW. For rings with $\varepsilon = \varepsilon_i$ according to analysis of dimensions the radiuses of the Mach disks y_{DMi}, related to radius y_1 of an annular edge and equal to radiuses of splitting points of coming SW, are the functions of M_0, γ and ε_i: $Y_{\varepsilon i} \equiv y_{DMi} / y_1 = F(M_\infty, \gamma, \varepsilon_i)$. On the other hand (fig. 10), for a ring with $\varepsilon_1 < \varepsilon_2$ the angle θ behind SW, increasing modulo, for some $y_2 < y_1$ becomes equal $-\varepsilon_2$. By virtue of the formula (2.1) it will take place when $y_2 / y_1 \approx \varepsilon_1^n / \varepsilon_2^n$. The SW point with ordinate $y = y_2$ is left under angle $\theta = -\varepsilon_2$ with a streamline, whose small initial site defines the SW shape and flow near it when $y < y_2$. The replacement of only this site contour by rectilinear generatrix with $\theta \approx -\varepsilon_2$ will give a new ring, practically not having changed the flow near SW up to a point of splitting and the size of a Mach disk y_{DM1}. Hence, when identical angles of an inclination of internal contours $\theta = -\varepsilon_2$ the ratios y_{DM2}/y_1 for the major ring and y_{DM1}/y_2 for the smaller one will coincide: $y_{DM2}/y_1 = y_{DM1}/y_2$. From here with account of the formula found above we shall receive

$$Y_{\varepsilon 2} / Y_{\varepsilon 1} = y_{DM2} / y_{DM1} \approx \varepsilon_2^n / \varepsilon_1^n. \tag{2.2}$$

Hence, $F(M_\infty, \gamma, \varepsilon) \approx f(M_\infty, \gamma)\varepsilon^n$ for $n > 2$.

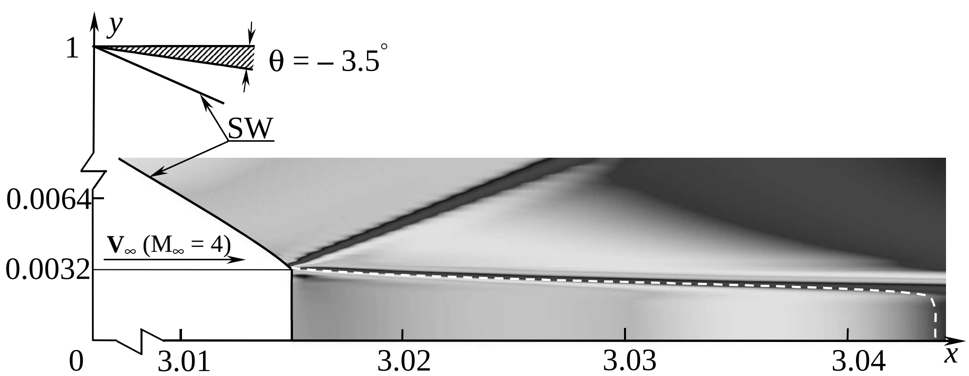

Fig. 13

For $M_\infty = 4$, $\varepsilon = 3.5^\circ$ Fig. 13 in a vicinity of the SW reflection point gives explicitly designed going to an axis of symmetry SW and a Mach disk close to normal shock, smeared (in a continuous field of the Mach numbers) reflected SW and tangential discontinuity and a dashed sonic line. Its close to horizontal site is the consequence of tangential discontinuity smearing, and almost vertical site is the real sonic line in a flow passed through the Mach disk. In this case the acceleration of a subsonic flow till supersonic velocity occurs per length about five diameters of a Mach disk, i.e. is not connected to a rarefaction wave which accompanies a flow of the annular channel back base or of its rectilinear contour point of turn. When $M_\infty = 4$, it is natural, that the same picture will be for anyone $\varepsilon < 3.5^\circ$. With increase of ε, since its some value, the acceleration of a subsonic flow behind a Mach disk will be influenced, and then will be defined by the mentioned rarefaction wave.

The values Y_ε of Mach disks, received by numerical integration of the Euler's equations for $M_\infty = 2, 2.5, 3, 4$ and $\varepsilon = 1.75^\circ$ и 3.5°, give the second and the third columns of the table, and their ratios are in the last column. When rather small values Y_ε the resulted data confirm agreed to the formula (2.2) fast decrease of a Mach disk radius with reduction of initial intensity SW. In [24] there is executed one calculation of originally weak shock reflection from an axis of symmetry for $M_\infty = 8.33$, using the through SW-calculation when not structured tend be smaller to an axis of symmetry grids. In it the curvilinear internal generatrixes of annular channels were designed along the streamlines of conic flow [10]. To initially weak shock the variant with $\varphi = 170.5^\circ$ was correspond when half-angle of inverse Mach cone equal 173.1°.

M_∞	ε, degree		$Y_{3.5}/Y_{1.75}$
	1.75	3.5	
2.0	0.0045	0.0210	4.67
2.5	0.0018	0.0091	5.00
3.0	0.0011	0.0055	5.05
4.0	0.0006	0.0032	5.52

When $M_\infty = 1.6$ for annular channels with $\varepsilon = 5^\circ$, 9° and 13° according to calculations of a method of characteristics [11] $y_{SW1}/y_1 \approx 0.05$, 0.34 and 0.83, and for their converged generatrixes the supersonic flows are designed in [11] when $0 \le x/y_1 \le X_F = 1.3$, 0.71 and 0.1. When replacement for $x/y_1 > X$ of inclined generatrixes by those, parallel to axis of symmetry, the numerical integration of the Euler's equations gives a flow of the two first channels with unattached SW. Only the third channel ($\varepsilon = 13^\circ$, $X = 0.1$) is flowed with SW, attached to a leading edge, and with generating of Mach disk which size ($Y_\varepsilon \approx 0.2$) is no more, but four times less than the size y_{SW1}/y_1 from [11]. The attached to the leading edges SW are obtained for two first channels only when appreciably smaller lengths of converged sites.

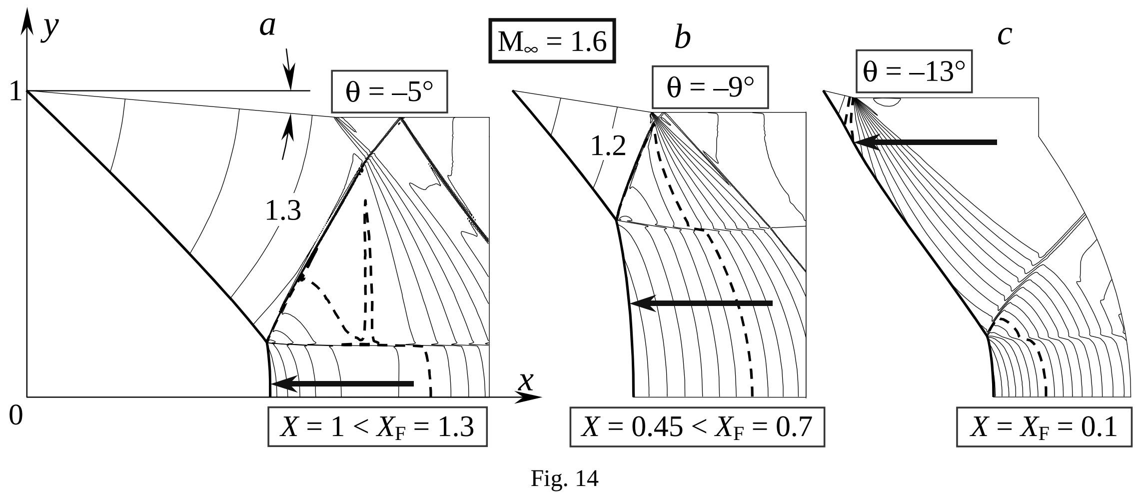

Fig. 14

For three channels SW, Mach disks, isomachs (with step $\Delta M = 0.05$) and in part coinciding with the smeared SW and tangential discontinuities sonic lines (are given by strokes) are submitted in Fig. 14 for three sets of values ε, X, Y_ε: 5°, 1, 0.18 (*a*), 9°, 0.45, 0.58 (*b*) и 13°, 0.1, 0.2 (*c*). In all three cases as against shown in Fig. 13 flows with originally weak SW the evolution of coming SW, the size of a Mach disk and acceleration of a subsonic flow behind it depend on the length of a converged site of the channel and from the wave of rarefaction going from a break in a point of joining with a cylindrical site. A consequence of it is the violation of suggesting in [1, 16] correlation of the Mach disk sizes with the value of y_{SW1}/y_1 (in Fig. 14 their values from [11] are shown by arrows). Actually such correlation there is, but only for rather primary weak SW, for which the size of a Mach disk is not influenced with the length of an inclined annular site. In agree with Fig. 14 the examples, designed in [11] and also in [12], did not satisfy to this requirement.

A pointed site of a sonic line in Fig. 14, *a* is confirmed by calculations for different computation grids. In Fig. 14, *c* the SW intensity with reduction *y* changes not monotonically.

The data collected in the table, formula (2.2) and the result [24], confirming the assumption stated for the first time by Courant and Fridrichs [1], explain the cause for which in experiments and calculations there can be observed the regular reflection from an axis of symmetry of initially weak SW. In the network of ideal gas here the scale factor takes place. In axisymmetric statement when through calculation of initially weak SW and when their explicit construction the Mach disk size is the minimum in times (in [17] twice), and more often – in orders less than cells size of usually used computation grids. In experiments the realization of strictly axisymmetric flows is impossible basically for many reasons: even for close to axisymmetric the model made on a turning lathe it is because of nonuniformity of an incident flow and small, but nonzero angle of its installation («angle of attack»). When flow from the nozzle of weak overextended supersonic jet to stated those there are added non-stationary and not axisymmetric disturbances arising when oscillations of a jet border. The effects of not axisymmetric and non-stationary amplify with approach the «top» of convergent SW, doing it near the expected «axis of symmetry» to be essential not axisymmetric and not conic.

For SW of appreciable initial intensity when the Mach disk is more the both: sizes of grid cells, and linear scales of indicated above and others experimental nonuniformities, the calculators and experimenters observe irregular reflection of SW from an axis of symmetry. As the size of a Mach disk according to the formula (2.2) with falling of SW intensity decreases very quickly then by virtue of the listed above reasons it disappears either because of the bad resolution of a numerical method, or because of violating of axisymmetry. In the both cases there appears an illusion of regular SW reflection from an axis of symmetry. At last, with reduction of initial SW intensity even in strict axisymmetric statement there will be an inevitable the necessity of the account of SW smearing effects because of the viscosity and then also of the discrete (atomic) structure of a flow.

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