EXCITATION OF GÖRTLER-INSTABILITY MODES IN CONCAVE-WALL BOUNDARY LAYER BY LONGITUDINAL FREESTREAM VORTICES

A.V. Ivanov, Y.S. Kachanov, D.A. Mischenko

Khristianovich Institute of Theoretical and Applied Mechanics SB RAS
630090, Novosibirsk, Russia

Introduction

When studying the problem of laminar-turbulent transition it is convenient to identify three main stages: (a) flow receptivity to different external perturbations, (b) linear stage of flow instability and (c) nonlinear flow instability and final formation of turbulent flow. The present experiments are devoted to investigations of the first stage applied to boundary layers developing over concave walls. The study of this case has a special interest due to presence of this flow instability to specific, three-dimensional disturbances. These disturbances usually look like a system of longitudinal counter-rotating vortices. The cause of this instability is associated with presence of wall-normal centrifugal forces due to mean-velocity gradient in the boundary layer. As a result, layers of high-speed liquid tend to move towards the wall, while low-speed layers tend to displace to the opposite direction. The described situation is typical, in general, for a broad class of shear flows with curved streamlines and known in literature as Görtler instability [1, 2]. Despite a great fundamental and practical importance, the Görtler instability still remains poorly investigated in general. This situation occurs due to extreme technical complexity of setting appropriate controlled experiments.

However, some new important results were obtained recently. A new, unsteady experimental approach to investigation of Görtler instability was developed and tested successfully by our research team. The linear stage of steady and unsteady instability was studied for the first time quantitatively. All main instability characteristics were obtained and several linear stability theories were verified quantitatively in a broad range of frequencies where the Görtler instability exists [3]. It was shown that the linear stability theory describes properly downstream evolution of Görtler vortices. The weakly-nonlinear stage of the instability was studied as well, and thresholds of nonlinearity were found out [4].

Besides the characteristics of the linear and nonlinear flow instability, initial amplitudes of boundary layer disturbances are also important for solution of the fundamental and practical problem of prediction of the laminar-turbulent transition location. These amplitudes are characterized by efficiency of various mechanisms of transformation of external disturbances into the boundary layer instability modes. These mechanisms correspond to the so-called receptivity problem. Its quantitative characteristics are described by the complex-valued receptivity coefficients. Until recently, the receptivity problem for the case of excitation of Görtler vortices has not been studied experimentally at all. Several experimental studies of receptivity of boundary layers on concave walls were carried out rather recently. They are: (1) receptivity to surface nonuniformities (roughnesses and vibrations) [5] and (2) receptivity to freestream vortices [6, 7]. These experiments were successful due to application of the new unsteady experimental approach mentioned above. For the first time, the corresponding receptivity coefficients for a broad range of frequencies and for two spanwise wavenumbers of surface nonuniformities (most dangerous according to the linear stability theory) were obtained. The study of the second problem has showed that the mechanisms of excitation of Görtler vortices due to scattering of 3D and 2D freestream vortices on 2D and 3D surface nonuniformities are inefficient. However, the same experiments have shown that longitudinal freestream vortices travelling near the boundary layer edge are able to lead to effective distributed excitation of Görtler vortices. It was shown that excited boundary layer
disturbances are able to grow downstream faster than in the corresponding linear stability problem and to be amplified even in regimes where the boundary layer is linearly stable to the Görtler vortices.

The procedure of obtaining the distributed receptivity coefficients is very complicated and connected with solution of inverse, mathematically ill-posed problem. Careful numerical analysis carried out in [7] showed that this ill-posed problem can be successfully solved by choosing physically correct form of the receptivity function. It was found that the indicated mechanism of distributed excitation of Görtler vortices in the boundary layer is the most efficient for disturbances with spanwise scales corresponding to the most dangerous (according to the linear stability theory) Görtler vortices.

Despite the first principal success, obtained in [6] and [7] data represent a first and rude (however a very important) result. Present study is devoted to detailed systematic experimental investigation of detected physical phenomena in a wide range of problem parameters.

1. Experimental model, the source of disturbances and a range of studied problem parameters

Experiments were carried out in low turbulent wind tunnel T-324 ITAM SB RAS at freestream velocity at the boundary layer edge of \( U_e = 9.18 \text{ m/s} \) in a zero-pressure-gradient boundary layer (close to the Blasius one) of a high-precision experimental model (Fig. 1) with concave wall having radius of curvature \( R = 8.37 \text{ m} \). Main measurements were performed with a hot-wire probe at excitation of fully controlled unsteady disturbances. The domain of the main measurements corresponds to Görtler numbers \( G^* = \left( U_e \delta_1 / \nu \right) \sqrt{\delta_1 / R} = 9 \text{ to } 20 \) (Here \( \delta_1 \) is the boundary layer displacement thickness and \( \nu \) is the kinematic viscosity of air). At the present coordinate system, \( x \) is a streamwise curvilinear axis (always parallel to the wall), \( y \) is the wall-normal coordinate and \( z \) is the spanwise one. The origin of the \( x \)- and \( z \)-axes is located in the middle of the plate leading edge.

Controlled unsteady freestream vortices with predominant streamwise vorticity component were generated in the freestream by a special disturbance source (Fig. 2): a wire vibrating at the specified frequency (with a diameter of \( D = 50 \text{ \mu m} \)) equipped with a localized micro-nonuniformity of special shape (with a thickness of about \( 100 \text{ \mu m} \) and length of \( 4.5 \text{ mm} \)). This wire was stretched parallel to the experimental model leading edge at a distance of \( 28 \text{ mm} \) upstream it. The freestream vortices generated by the vibrating wire propagated along the boundary-layer outer edge. Careful
Fig. 2 Sketch of generation of 3D freestream vortices by vibrating wire with local nonuniformity.

investigations have shown that these freestream disturbances led to efficient distributed excitation in the concave-wall boundary layer of unsteady Görtler vortices with a broad spanwise-wavenumber spectrum (Sec. 2).

The experiments were carried out at three frequencies of the freestream vortices \( f = 8, 13 \) and \( 17 \) Hz (dimensionless frequency parameter \( F = 2\pi f U_c/V = 9.07, 14.75 \) and \( 19.29 \)). These frequencies correspond to three qualitatively different regimes of evolution of Görtler vortices (according to the linear stability theory) [3]. The investigations are carried out in a wide range of dimensionless spanwise scales \( \lambda = (U_c \lambda_v V)/(\delta_c R) = 149-774 \) (here \( \lambda_z = 8 \) to \( 24 \) mm is the dimensional spanwise wavelength of Görtler vortices).

2. Excited boundary layer disturbances: the Görtler vortices

The boundary layer disturbances excited by means of the distributed receptivity mechanism were found to be localized in the spanwise direction in the vicinity of controlled 3D longitudinal freestream vortices. Typical spanwise profiles of boundary layer disturbance amplitudes (left axis) and phases (right axis) are shown in Fig 3a for the streamwise component of the velocity disturbance vector. Measurements are carried out at a wall-normal distance corresponding approximately to the amplitude maxima of unsteady Görtler vortices, i.e. at \( U(y)/U_c = 0.6 \) [3] (here \( U \) is the streamwise mean flow velocity component). The measured perturbations correspond to a spanwise localized wave packet. The positions of their amplitude maxima corresponds to the location of couple of the controlled 3D (longitudinal) freestream vortices generated by the nonuniformity mounted on the vibrating wire (see Sec. 1).

Figure 3b shows wall-normal profiles of disturbance amplitudes and phases measured far from the location of the vibrating wire nonuniformity in the spanwise direction, i.e. in the region where the freestream vortices are two-dimensional (location I in Fig. 3a). The vertical dash-dotted line shows approximate location of the boundary layer edge (with \( y = \delta \)). Far from the uniformity, the vibrating wire generates two-dimensional antisymmetric vortex street. Wall-normal profile of the vortex street amplitude has two humps oscillating in antiphase. These freestream vortices do not lead to excitation of any boundary-layer disturbances; their amplitudes decay monotonously inside the boundary layer in the wall-normal direction.

Completely different picture is observed in the spanwise area corresponding to the position of the longitudinal freestream vortices generated by the vibrating-wire nonuniformity. Figure 3c display the corresponding profiles, measured at location II shown in Fig. 3a. Excitation of some boundary layer disturbances of rather large amplitudes is clearly seen here. As a result, an additional maximum arises in the wall-region of the amplitude profile. (The disturbance amplitude is greater in the wall region than the amplitude of the freestream vortex street by a factor of 2 approximately.) The boundary layer disturbance phases decay practically in a linear way with a wall-normal distance. In addition to experimental points, the results of calculations based on the locally-parallel linear stability theory (LST) are shown there with thick solid lines. (These calculations have been carried out by means of a programs of A.V. Boiko [3] for the case of the most dangerous Görtler vortices with the spanwise wavelength \( \lambda_z = 10 \) mm, \( \Lambda = 208 \).) The positions
of the experimental and theoretical amplitude maxima, as well as the shapes of the amplitude and phase profiles are in good correlation with each other in the near-wall region. Similar results are seen in Fig. 3d at the wall-normal amplitude and phase profiles measured at the spanwise location III shown in Fig. 3a, where the main amplitude maximum of the wave-packet disturbance is observed. (At this spanwise position, the amplitudes of the boundary layer disturbances several times greater than the amplitudes of the freestream vortices.)

![Graphs showing amplitude and phase profiles](image)

Fig. 3 Amplitude and phase profiles of boundary layer disturbances.

- a – typical spanwise profiles measured at \( U(y)/U_e = 0.6 \).
- b, c, d – wall-normal profiles, measured at spanwise locations I, II and III shown in plot a. Bold solid lines display results of LST calculations carried out for most growing Görtler vortices with \( \lambda_z = 10 \) mm \( (\Lambda = 208) \). Vertical dash-dotted line is approximate position of boundary layer edge.

\( f = 8 \) Hz, \( F = 9.07 \), \( x = 636 \) mm, \( G = 14 \).

Frequency-wavenumber spectra of the boundary layer disturbances shows that the perturbations have the largest spectral amplitudes at the spanwise wavelengths of 8 to 14 mm \( (\Lambda = 149 \div 345) \). According to the linear stability theory, the Görtler vortices with exactly these spanwise scales are the most quickly growing in boundary layer. The phase velocities of the boundary layer disturbances are close to 0.65 of the freestream speed, i.e. close to characteristic values of the phase velocities obtained for the linear unsteady Görtler vortices [3]. Thus, all obtained data display that the boundary layer disturbances excited by of the present distributed receptivity mechanism correspond to the unsteady Görtler vortices. (The results described above in Sec. 2 are in a good agreement with those obtained earlier in experiments [6, 7].)

The studies have shown that the mechanisms of the linear instability and distributed vortex receptivity of the boundary layer compete with each other. Figure 4 display streamwise distributions of disturbance amplitudes, normalized by the first point amplitude, (in logarithmic scale a,b,c) and disturbance phases (d) of Görtler vortices at three investigated frequencies $f = 8$ (a, d), 13 (b, d) and 17 Hz (c, d) (the corresponding dimensionless frequency parameters are $F = 9.07$, 14.75 and 19.29 correspondingly) for three spanwise scales $\lambda_z = 10, 15$ (only for the case of $f = 17$ Hz) and 20 mm (the corresponding dimensionless spanwise scales are $\Lambda = 208, 383$ and 590). Points denote the experimental data, lines show the LST results. The values of local Görtler numbers are indicated on an additional upper axis. Detailed studies [3] have shown that according to the linear stability theory, the most quickly growing disturbances are the Görtler vortices having spanwise scales of 8 to 14 mm. Further increase of the spanwise scale leads to decay of the growth rates of Görtler vortices at all frequencies and the unsteady vortices of larger spanwise scales (with $F \geq 5.5$) are attenuate typically downstream. As was shown in [3], the steady Görtler vortices are the most dangerous according to the linear stability theory. The larger is the frequency the lower are the growth rates of Görtler modes of all spanwise scales. Due to this, the boundary layer becomes linearly stable with respect to unsteady Görtler vortices of all spanwise scales having frequency parameters $F > 23$.

![Fig. 4 Comparison of experimental streamwise evolution of unsteady Görtler vortex amplitudes (normalized at the first point) (a, b, c) and phases (d) with that predicted by LST.](image)

At low frequencies (about 8 Hz), the evolution of Görtler modes is mainly determined by the linear-instability laws. Although the presence of the longitudinal freestream vortices leads to a rather efficient excitation of Görtler modes, the distributed receptivity mechanism is insufficient at these frequencies for a significant change of the Görtler vortex growth rates. Therefore, the amplification of the boundary layer disturbances is similar to that provided by the linear instability mechanism and the spatial increments are close to those predicted by the LST. When the frequency increases and, hence, the linear-instability mechanism gets weaker, the picture changes...
significantly. In the region of moderate frequencies (about 13 Hz) the influence of the distributed receptivity mechanism on the disturbance amplification becomes quite substantial and the boundary layer disturbance amplitudes grow considerably faster then this is predicted by the LST (Fig. 4b,d). (Note that in our previous experiments [3] a very good quantitative agreement of amplification curves of disturbance amplitudes and phases with the Görtler linear stability theory was observed in a broad range of parameters of the problem.) As seen in Fig. 4cd, at high frequencies (about 17 Hz), the investigated receptivity mechanism is able to play a predominant role in the streamwise evolution of the Görtler instability modes. In particular, this mechanism is able to lead to amplification of neutrally stable boundary layer disturbances (at $\lambda_z = 15$ mm) and even of linearly stable ones (at $\lambda_z = 20$ mm).

Thus, in many practical cases occurred at high freestream turbulent level at real conditions, the amplitudes of high-frequency Görtler vortices can be very significant under the influence of the distributed vortex receptivity mechanism discussed above. These vortices are able to arise, grow intensively, and influence significantly (due to the nonlinear effects) the streamwise evolution of the Görtler modes in a broad frequency-wavenumber spectrum. As the result, the investigated receptivity mechanism is able to lead to earlier laminar-turbulent transition of laminar boundary layers.

4. Distributed receptivity coefficients

The experimental data obtained allowed to us to estimate quantitative characteristics of investigated phenomena. They were obtained in a same way as in earlier works [6, 7]. The differential equation (1) describes evolution of complex amplitudes of unsteady boundary-layer disturbances of the streamwise velocity component associated with Görtler vortices $\overline{B}^d$ with the streamwise coordinate $x$:

$$
\frac{d\overline{B}^d(x, y_m)}{dx} = i\overline{\alpha}(x)\overline{B}^d(x, y_m) + \overline{B}_v(x, y) \bigg|_{y=d} \overline{G}^d_v(x)
$$

(1)

Two mechanisms are responsible for evolution of the Görtler vortex amplitudes and phases in the boundary layer: $I$ is the mechanism of boundary layer linear instability to this disturbances ($\overline{\alpha}(x)$ is the complex streamwise wavenumber), and $II$ is the mechanism of boundary layer distributed receptivity to freestream longitudinal vortices ($\overline{B}_v(x, y) \bigg|_{y=d}$ are complex amplitudes of this vortices, measured at the boundary-layer edge; $\overline{G}^d_v(x)$ is the seeking complex receptivity function).

Equation (1) can be considered as a definition of the distributed receptivity coefficients. It has an analytical solution:

$$
\overline{B}^d(x^*) = e^{\overline{\alpha}(x^*)} \left\{ \int_0^{x^*} \overline{B}_s(s)\overline{G}^d_v(s)e^{-\overline{\alpha}(s)}ds + \overline{B}_0^d \right\}, \quad \overline{A}(x^*) = \int_0^{x^*} \overline{\alpha}(s)ds
$$

(2)

For convenience, the origin of the streamwise coordinate is located in (2) at the first point of the region of main measurements (the $x^*$-coordinate). $\overline{B}_0^d$ is an “initial” complex amplitude of boundary layer disturbances (observed at $x^* = 0$).

For each disturbance frequency and spanwise scale, the unknown complex-valued function $\overline{G}^d_v(x^*)$ and values $\overline{B}_0^d$ can be found by means of approximation of the experimental distributions $\overline{B}^d(x^*)$ by the analytical solution (2). In the present work, $\overline{\alpha}(x)$ was taken from the LST-calculations carried out for the particular experimental conditions. As was shown in [3], our experimental data obtained on the linear Görtler instability problem are always in a very good quantitative agreement with the LST-calculations.
The described approximations were carried out by means of MATLAB optimization tools (gradient, simplex, and genetic algorithms) using two different optimization criteria. As was shown earlier [7], the receptivity functions are able to be well approximated by complex exponents with amplitude decaying downstream.

A typical example of the described approximation of the experimental (symbols) amplitudes (left) and phases (right) by the corresponding analytical solutions are shown in Fig. 5. Curves of different colors correspond to analytical solutions obtained by using different combinations of algorithms and criteria of optimization (10 curves in total). As can be seen, all curves are in a good agreement with experimental data and practically coincide with each other.

![Figure 5 Example of approximation of experimental amplification curves (symbols) of Görtler vortex amplitudes (left) and phases (right) by analytical solutions (colored lines). These solutions are obtained by means of various algorithms and criteria of optimization. \( \lambda_z = 10 \text{ mm}, \Lambda = 208, f = 13 \text{ Hz} F = 14.75 \).](image)

The most important result of our study is presented in Figs. 6 and 7 as amplitudes and phases of the complex-valued receptivity coefficients versus spanwise scale obtained for three frequencies at two streamwise positions. Different curves denote results obtained by using different algorithms and criteria of the approximation procedure. Deviations of these curves from each other can be interpreted as a rough estimation of a confidence interval of the obtained data. As can be seen, the results obtained by means of different methods are very close to each other, in the most of cases.

Figure 6 shows the most important (in the physical sense) result: the receptivity amplitudes measured in the beginning (left) and in the end (right) of the region of measurements. These results are normalized by the boundary-layer displacement thickness \( \delta_1 \) (within the region of measurements \( \delta_1 \) increases by a factor of 1.4).

As can be seen in Fig. 6, the amplitudes of the receptivity coefficients have a common maximum in the region around \( \Lambda = 150 \) to 350 (\( \lambda_z \approx 8 \) to 14 mm). According to linear stability theory, the Görtler modes of these spanwise scales are the most amplified. Thus, the mechanisms of the boundary-layer linear instability and the boundary-layer distributed vortex receptivity are able not only compete with each other but also to enhance each other. Absolute values of amplitudes of the distributed receptivity coefficients decrease with frequency. However, in the area of low and moderate frequencies (around \( f = 13 \) Hz, \( F = 14.75 \) and less) this decrease is insignificant for vortices with spanwise scales \( \Lambda = 150 \) to 350 (i.e. for the most linearly dangerous Görtler disturbances). The downstream decay of the distributed receptivity amplitudes is enhanced with frequency. At the same time, in the area indicated above, this acceleration is significantly weaker than that of the Görtler vortices having greater spanwise scales.
The vortex receptivity phases are presented in Fig. 7 versus spanwise scale also for three frequencies for the beginning of the region of measurements only. The distributions are seen to be smooth (excluding a jump by 180 degrees in Fig. 7a). Note also that different methods and criteria of the approximation give usually almost the same results. This suggests a rather high reliability of the obtained distributions.

![Fig. 6 Amplitudes of distributed vortex receptivity coefficients versus spanwise scale for three frequencies and two streamwise positions (Görtler numbers). Different curves are obtained by different algorithms and criteria of optimization.](image)

In general, the results of the present study show that observed and investigated mechanism of excitation of unsteady Görtler instability modes in a concave-wall boundary layer can be regarded as one of the most significant "natural" ways for their generation in various aerodynamic applications at conditions of enhanced freestream turbulence. The studied physical phenomenon may be important for solution of the fundamental problem of the laminar-turbulent transition prediction in the cases indicated above.
Fig. 7 Phases of distributed receptivity coefficients versus spanwise scale obtained for \( x^* = 163 \) mm for three frequencies.

**MAIN RESULTS**

1. It is found that the studied mechanism of distributed receptivity of the boundary layer on concave wall is able to excite Görtler modes effectively. This mechanism is also able to modify significantly the streamwise evolution of these modes and transform attenuating Görtler vortices (due to the linear instability mechanism) into amplified ones.

2. The mechanisms of boundary layer linear instability and distributed receptivity can be in a competition with each other. At low frequencies the evolution of boundary layer disturbances is determined mainly by the linear instability lows, while at higher frequencies, contribution of the receptivity mechanism to evolution of Görtler modes becomes significant due to weakening of the linear instability mechanism. Therefore, at the highest studied frequency the evolution of the distributedly excited Görtler vortices is almost completely determined by the receptivity mechanism.

3. It is found that the amplitudes of the distributed vortical receptivity coefficients have their maxima in a range of dimensionless spanwise scales corresponding to Görtler modes amplifying most rapidly by the linear instability mechanism (\( \Lambda = 150-350 \)). In this sense, the mechanisms of the distributed receptivity and of linear instability are able to intensify each other.

4. It is shown that amplitudes of the receptivity coefficients decay downstream with a rate increasing with frequency.

5. The investigated receptivity coefficients are found to attenuate with frequency although this attenuation is rather weak at frequencies lower then about 13 Hz (\( F = 14.75 \)).

This work is supported by the Russian Foundation for Basic Research (grant № 12-01-31211).
REFERENCES