ON NONLINEAR INTERACTION OF CONTROLLED DISTURBANCES IN 3D SUPERSONIC BOUNDARY LAYER

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Introduction. The transition process in boundary layers is mainly characterized by three stages: receptivity, growth and breakdown of disturbances. The stage of growth, in turn, can be divided into regions of the linear and nonlinear development of perturbations. The process of amplification of the initially small unstable disturbances is described well by the linear theory of hydrodynamic stability. At some position nonlinear effects become dominant and lead to breakdown to turbulence. One of the most common and often realized mechanism of the initial stage of nonlinearity is subharmonic three-wave resonance.

Most theoretical and experimental results on nonlinear interactions of disturbances have been obtained for two-dimensional subsonic and supersonic boundary layers. Application of weakly nonlinear theory [1] yielded to achieve a sufficient interpretation of the experimental results [2] on the transition process in subsonic case. For supersonic flows similar analysis was also performed, and it was possible by performing experimental studies [3, 4] where controlled disturbance technique was used. Such technique of introduction artificial disturbances in the boundary layer allowed conducting experimental research of the pre-turbulent states of the flow and stimulating the construction of theoretical models [5, 6].

The problem of the transition to turbulence in three-dimensional (3D) boundary layers is more complicated due to the presence of a cross-flow and existence of different instability modes strongly dependent on environment conditions. Very few theoretical and experimental investigations of nonlinear interactions in 3D compressible boundary layer have been fulfilled up to date. They are commonly confined to the linear stage [7, 8] where good quantitative agreement with experimental observations and numerical simulations was achieved [9]. Nevertheless the principal possibility of three-wave resonant interaction in swept-wing boundary layer is known over a long period of time [10]. The goal of the present study is to study nonlinear wave processes and to confirm the existence of subharmonic resonance in swept wing supersonic boundary layer at Mach 2 using controlled disturbance technique.

Experimental equipment and data processing. The experiments were conducted in the T-325 low noise supersonic wind tunnel of ITAM SB RAS at Mach 2 and unit Reynolds number $Re_1 = 5.0 \times 10^6$ / m. To determine the flow parameters and to perform experiments the wind tunnel was equipped with an automated measuring system.

The swept wing model used in the experiments had a sharp leading edge (with blunting radius of 0.2 mm) and a sweep angle of 45°. The model had a thin (3%) planar-convex airfoil with a maximum thickness of 12 mm. The radius of curvature of the test surface of the model was about 4000 mm and the bottom surface was flat. Some information about the swept wing model and the coordinate systems are shown in Figure 1. The model was rigidly fixed in the central plane of the test section of the tunnel at approximately zero angle of attack. A source of localized artificial disturbances was built in the model [11]. Controlled pulsations have been generated by high frequency glow discharge in chamber and they were injected into boundary layer through an aperture, 0.4 mm in diameter, in the working surface of the model. Coordinates for source aperture were $x = 56.6 \pm 0.3$ mm from the leading edge of the model and $z = 0$ mm, which coincides with the center line of symmetry of the model. Pure sine wave generator and an amplifier with transformer output were used to produce high voltage at frequency of 10 kHz that allow exciting artificial perturbations at frequencies of 10 and 20 kHz in the boundary layer.
The disturbances in the flow were measured with a constant-temperature hot-wire anemometer. Probes of tungsten wire, 10 micron in diameter and 1.6 mm long, were employed. The probe moved in the three coordinates \( x, y, z \), and the measurement accuracy in the \( x \)- and \( z \)-coordinates was to within 0.1 mm and 0.01 mm in \( y \). The probe overheat ratio was taken to be about 0.8 and 90 to 95% of the measured disturbances were the mass flow rate fluctuations.

The fluctuation signal from the anemometer output was digitized using a 12-digit analog-to-digital converter (ADC). The ADC discretization frequency was 750 kHz. The length of the digital oscillograms recorded in the experiments amounted to 65536 points. The pulsation measurements were synchronized with the generator giving the frequency of the disturbances introduced. At any spatial point four measurements were carried out. In analyzing the natural fluctuation development the data from the full realizations were processed. To separate out the controlled disturbances the averaging over four realizations was carried out.

The resulting fluctuation oscillograms normalized by the mean voltage of the hot wire anemometer at each spatial measurement point were transformed in the mass flow rate fluctuations according to the relation

\[
m(x', z', t) = \frac{e'(x', z', t)}{Q \times E(x', z')}
\]

Here, \( e'(x', z', t) \) is the fluctuation signal at the anemometer output, \( E(x', z') \) is the mean voltage at the anemometer output, and \( Q \) is the anemometer probe sensitivity to the mass flow rate fluctuations. The frequency-wave disturbance spectra at a fixed coordinate \( x' \) were determined using the double discrete Fourier transformation (DFT) in time and the transverse \( z' \)-coordinate

\[
m_{f\beta'} = \frac{\sqrt{2}}{T \delta_n} \sum_{j,k} m(x'_j, t_k) \exp[i(\beta'z'_j - \omega t_k)] \Delta t \Delta z'_j,
\]

where \( T \) – time-length of realization; \( \delta_n \) – the boundary layer thickness scale for normalizing the spectra in \( \beta' \); \( \delta_n = 1 \) mm.

After the double Fourier transformation the wave amplitude and phase were determined as

\[
A_{f\beta'} = \sqrt{\Re^2(m_{f\beta'}) + \Im^2(m_{f\beta'})},
\]

\[
\Phi_{f\beta'} = \arctg[-\Im(m_{f\beta'})/\Re(m_{f\beta'})].
\]

The wavenumbers \( \alpha_r \) of the longitudinal component of the wave vector were determined from the formula

\[
\alpha_r = \frac{\Delta \Phi_{f\beta'}}{\Delta x},
\]

where \( \Delta x \) - the distance between the measured sections in the axis \( x \) direction. The wavenumber \( \alpha' \), was determined from the relation

\[
\alpha' = \frac{\alpha_r}{\cos(\alpha)} - \beta' \tan(\alpha),
\]

where \( \alpha \) is the wing sweep angle.
Experimental results and discussion. First of all analysis of the development of natural disturbances is required. It is important for estimating the space regions of linear and nonlinear growth of the perturbations. In accordance with this, positions of the measuring sections are chosen. The dependence of the integral fluctuations \( <m> \) and the mass flow rate \( \rho U \) downstream are shown in Figure 2. Here the longitudinal coordinate \( x_l \) is measured from the leading edge of the model. The measurements were performed in the boundary layer at a constant mass flow rate \( \rho U \) level equal to about 70% of the value in the external flow.

The positions of the linear and nonlinear regions of the disturbance development are determined from the statistical analysis, in which the presence of the Gaussian distribution of the probability density for the fluctuation amplitude indicates the linearity of the process, while a deviation from the normal distribution points out on the nonlinearity of the process [12]. It was obtained that the natural disturbances develop linearly up to the coordinate \( x_l \approx 130 \) mm or the Reynolds number, corresponding to this coordinate \( Re \approx 0.65 \times 10^6 \). At high Reynolds numbers natural disturbances developed nonlinearly, since their distribution densities for the fluctuation amplitude considerably deviate from the normal distribution law. In accordance with this experimental data were obtained at \( x_c = 60, 70, 80 \) mm from the source of controlled disturbances. Moreover it was found that a fundamental frequency wave (20 kHz) and its subharmonic wave (10 kHz) belong to the range of the unstable in linear sense waves [8].

Figure 3 presents distributions of amplitude and phase disturbances on the \( z' \)-coordinate at 10 kHz and 20 kHz. Disturbances at frequency 20 kHz were developed downstream as the packet of eigenwaves of the supersonic boundary layer. As previously obtained [8] the wave train with fundamental frequency 20 kHz becomes smeared in the transverse direction, while its center is displaced toward positive values of the \( z' \)-coordinate and the phase increases almost linearly. Downstream evolution at frequency 10 kHz disturbances gets additional features. There are amplitude growth in the region where in the case of the linear development of the subharmonical wave amplitude is negligible [13]. Here the indicated amplitude is even higher than those corresponding to a linear development of the packet with frequency 10 kHz. Moreover spatial regions of the basic packet and additional field perturbation are shift in phase distributions of disturbances.

The \( \beta' \)-spectra of waves are shown in Figure 4 for the fundamental frequency and subharmonic frequency. Wave spectra at frequency 20 kHz corresponds to the linearly growing wave, which have a maximum at \( \beta' = 1.06 \) rad/mm [8, 13]. Another character of \( \beta' \)-spectra at 10 kHz are discovered. There are several peaks in \( \beta' \)-spectra amplitude which exceed measurement mistake. If maxima at \( \beta' = 0.8 \) rad/mm are usual to eigenwaves of the swept wing supersonic boundary layer at fre-
Section 2: Stability, Turbulence, Separation

Frequency 10 kHz [13] then appearance of increasing waves with wave numbers $\beta' = 0.19; 1.6 \text{ rad/mm}$ is unexpected with the linear stability theory viewpoint for the boundary layer at our experimental conditions. At indicated wave numbers the phase is nearly constant in phase spectra when $\beta'$ increases. This means that the wave packet corresponding to this spectra section is localized in the region close to $z' = 0$.

![Phase Spectra](image1)

![Phase Spectra](image2)

**Fig. 3.** The amplitude and phase of artificial disturbances at $x_c = 60, 70, 80 \text{ mm}$.

![Amplitude Spectra](image3)

**Fig. 4.** The amplitude wave spectra of artificial disturbances at $x_c = 60, 70, 80 \text{ mm}$.

To explain the appearance of additional peaks in $\beta'$-spectra at 10 kHz estimations of longitudinal wavenumbers $\alpha_r'(\beta')$ and the verification of implementation of subharmonic resonance conditions [14] were made. Three-wave resonant interaction conditions can be written as

$$f_1 + f_2 = f_3, \quad \alpha_{r1} + \alpha_{r2} = \alpha_{r3}, \quad \beta_1 + \beta_2 = \beta_3.$$
where subscripts correspond to three components of the triplet. Here subscript 3 refers to the fundamental wave (20 kHz) as the wave responsible to nonlinear interaction. Two others subscripts refer to subharmonic waves at frequency 10 kHz. Resonance conditions for wavenumbers were checked on maxima in amplitude $\beta'$-spectra. In our case wave triplet can be find at $\beta' = 0.19; 0.8$ rad/mm at frequency 10 kHz and $\beta' = 1.06$ rad/mm at 20 kHz. The data presented in Figure 5 shows the carrying-out of subharmonic resonance conditions for indicated triplet. The presence of the peak at $\beta' = 1.6$ rad/mm for subharmonic wave (10 kHz) remains under consideration. We are planning to continue the investigation to clarify the nonlinear processes in the swept wing supersonic boundary layer transition.

$$
\begin{array}{ccc}
\hline
f, \text{kHz} & \alpha'_r, \text{rad/mm} & \beta', \text{rad/mm} \\
10.00 & 0.11 & 0.19 \\
10.00 & -0.52 & 0.8 \\
20.00 & -0.48 & 1.06 \\
\hline
\end{array}
$$

![Graph showing $\alpha'_r, \text{rad/mm}$ vs $\beta', \text{rad/mm}$]

**Fig. 5.** The carrying out of conditions of subharmonic resonance.

**Conclusions.** An experimental study of nonlinear interactions of controlled disturbances in supersonic boundary layer on swept wing at Mach 2 was carried out. The position of nonlinear stage of natural disturbance evolution was defined. In downstream direction nonlinearity of the controlled disturbance evolution was detected and appearance of additional peaks in $\beta'$-spectra at subharmonic frequency can be explained in terms of subharmonic resonance. It was experimentally demonstrated a possibility of parametric amplification of stable in the linear sense disturbances in a wide range of spanwise wavenumbers in the 3D supersonic boundary layer.

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**REFERENCES**


Section 2: Stability, Turbulence, Separation


