CONSTRUCTION OF CONFIGURATION FOR A BODY WITH HIGH LIFT-DRAG RATIO MINIMIZING THE HEAT FLUX TO ITS SIDE SURFACE

V.I. Lapygin, A.B. Gorshkov, D.M. Fofonov, T.V. Sazonova, V.A. Mikhalin

Central Research Institute of Machine Building,
141070, Korolev, Moscow Reg., Russia

Introduction

High supersonic gas flow is characterized by high heat content (enthalpy) and high heat flux to a body surface; this should be remembered during construction of an optimal body shape. In particular, the mandatory requirement on optimal configuration in high-enthalpy supersonic flow is bluntness of nose and leading edges. Bluntness shape of axisymmetric and flat bodies of minimal drag is studied in [1-3], which results can be used for determining a shape of blunted nose and leading edges of three-dimensional configurations.

The body shape of maximal lift-drag ratio (LDR) in hypersonic flow is studied in details [4], and the results obtained do not include heat application to a surface of such body.

A problem on limitation of heat flux to a body surface can be formulated as determination of a body shape, which either minimize heat flux at given LDR value $K$ or realizing maximal LDR at given heat flux.

Algorithms for calculating force and heat loads that are used in course of computations should be both sufficiently simple for numerical realization and simulate correctly the real physical processes in high temperature gas flow. In this connection the tangent wedge method is used below for determining the force loading, and simulation of heat loading is going using the Reynolds analogy, which validity will be discussed by the example of a flow around a delta wing.

1. Hypersonic flow around delta wing

Let’s consider a hypersonic viscous gas flow around a delta with blunted nose and leading edges. The Navier – Stokes equations written in arbitrary curvilinear coordinates for non-equilibrium gas flow are use as mathematical model:

$$\frac{\partial \tilde{Q}}{\partial t} + \frac{\partial (\tilde{E} - \tilde{E}_v)}{\partial \xi} + \frac{\partial (\tilde{F} - \tilde{F}_v)}{\partial \eta} + \frac{\partial (\tilde{G} - \tilde{G}_v)}{\partial \zeta} = \tilde{S}$$

$$\xi = \xi(x, y, z, t), \quad \eta = \eta(x, y, z, t)$$
$$\zeta = \zeta(x, y, z, t), \quad \tau = t$$

$$\tilde{Q} = J^{-1}Q; \quad \tilde{S} = J^{-1}S$$
$$\tilde{E} = J^{-1}(\xi, Q + \xi, E + \xi, F + \xi, G)$$
$$\tilde{F} = J^{-1}(\eta, Q + \eta, E + \eta, F + \eta, G)$$
$$\tilde{G} = J^{-1}(\zeta, Q + \zeta, E + \zeta, F + \zeta, G)$$
$$\tilde{E}_v = J^{-1}(\xi, E_v + \xi, F_v + \xi, G_v)$$
$$\tilde{F}_v = J^{-1}(\eta, E_v + \eta, F_v + \eta, G_v)$$
$$\tilde{G}_v = J^{-1}(\zeta, E_v + \zeta, F_v + \zeta, G_v)$$

Section No. 3: Aerogasdynamics of Internal and External Flows

\[
Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{pmatrix}, \quad 
E = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ \rho u w \\ (E + p)u \end{pmatrix}, \quad 
F = \begin{pmatrix} \rho v \\ \rho v u \\ \rho v^2 + p \\ \rho v w \\ (E + p)v \end{pmatrix}, \quad 
G = \begin{pmatrix} \rho w \\ \rho w u \\ \rho w v \\ \rho w^2 + p \\ (E + p)w \end{pmatrix}
\]

\[
E_v = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \\ m_x \end{pmatrix}, \quad 
F_v = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \\ m_y \end{pmatrix}, \quad 
G_v = \begin{pmatrix} 0 \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{zz} \\ m_z \end{pmatrix}, \quad 
S = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega \end{pmatrix}
\]

\[
m_x = u \tau_{xx} + v \tau_{xy} + w \tau_{xz} - q_x,
\]

\[
m_y = u \tau_{xy} + v \tau_{yy} + w \tau_{yz} - q_y,
\]

\[
m_z = u \tau_{xz} + v \tau_{yz} + w \tau_{zz} - q_z.
\]

Total energy is written as a sum:

\[
e = \sum_i \rho_i e_i + \sum_i \rho_i h_{f,i} + \rho \frac{u^2 + v^2 + w^2}{2}
\]

\[
e_i = C_{v,f}^i T + e_i^{vib}(T) + e_i^{el}(T), \quad h_i = e_i + p_i / \rho_i.
\]

Gas mixture pressure is defined by the Dalton law:

\[
p = \sum p_i = \sum \frac{\rho_i RT}{M_i}.
\]

Numerical algorithm of the solution is based on the Gauss-Seidel iteration (LV-SGS) [5]. Parameters of the flow around the wing are calculated by the method [6] assuming equilibrium-radiation temperature of the surface \(q_w = 0.8\sigma T_{w}^4\), here \(\sigma\) – the Boltzmann constant.

With this mathematical model the non-equilibrium air flow around the wing with wedge-like profile and flat lower surface (Fig. 1a) was calculated for \(V = 5\) km/s, \(Re_l = 10^6\) and the range of angle of attack \(\alpha\) from 0 to 14°. The bluntness radius of the nose and leading edges were equal to \(r = 0.01l\), where \(l\) – root chord length, sweep angle of the leading edge \(\chi = 70^\circ\), volume \(W = 0.0195l^3\).

Calculation results are plotted against the angle of attack in Fig. 2 as dependencies of heat flux \(Q\) to the nose and leading edges, to the lower (windward) and upper (leeside) surfaces, and also total heat flux. For examined range of the angles of attack the heat flux to the nose and leading edges does not depend, in practice, on the angle of attack, whereas the heat flux to the side surface of the body increases with increase of the angle of attack.
With the angle of attack $\alpha \approx \alpha^*$, where $\alpha^*$ corresponds to maximal lift-drag ratio $K_{\text{max}} = K(\alpha^*)$, heat flux to the windward side is several times higher as compared with lee-side, and total heat flux to the body surface, along with LDR value, is defined by the shape of this surface.

\[ \frac{\text{St}}{\text{v}} = q_{w} / \rho_{\infty} V_{\infty}^{2}, \quad C_f = \frac{2 \tau_w}{\rho_{\infty} V_{\infty}^{2}}, \]

where $\tau_w$ -- friction stress on the wing surface, $\rho_{\infty}$, $V_{\infty}$ -- oncoming flow density and velocity.
Section No. 3: Aerodynamics of Internal and External Flows

It is seen in the plots at Fig.3 that excluding the plane of symmetry and vicinity of leading edge bluntness, the ratio St/Cf varies weakly and is close to a constant value (k1 = 0.53 for windward side and k2 = 0.47 for lee-side surface). In other words at the most part of the wing surface the Stanton number St is proportional to Cf coefficient, i.e. The Reynolds analogy is valid [7].

It is noted that the analogy is valid for flows around wings with diamond and lentiform cross sections too.

2. Optimal configuration

Let’s consider supersonic flow around a body, which leeside is described by the equation \( y + f_1(x,z) = 0 \) and windward side – by equation \( y + f_2(x,z) = 0 \). X-axis of the right rectangular coordinates makes an angle \( \alpha \) with oncoming flow velocity vector \( \vec{V} \), and Y-axis is pointing up.

In the body-axis coordinates the coefficient of aerodynamic forces \( C_y, C_x \) and quantity of heat \( Q \) to the body side surface are written in the following form:

\[
S_b C_y = \iint_S (C_{p1} - C_{p2} - C_{f1}/\Delta_{11} - C_{f1}/\Delta_{12}) dxdz
\]  

\[
S_b C_x = \iint_S (C_{p1} u_i - C_{p2} u_i + C_{f2} \Delta_{11} + C_{f2} \Delta_{12}) dxdz
\]

\[
\overline{Q} = \frac{QS_b}{\rho V^3} = \iint_S (St_1 \Delta_1 + St_2 \Delta_2) dxdz
\]

\[
S_b = \iint_S dxdz
\]

\[
u_i = \frac{\partial f_i}{\partial x}, w_j = \frac{\partial f_j}{\partial z}, \Delta_1 = 1 + u_i^2 + w_j^2, \Delta_2 = 1 + w_j^2
\]

\[
C_{pi} = C_p(\xi_i, \xi_j) = (\vec{P}, \vec{V}) = (u_j \cos \alpha + \sin \alpha) / \Delta_j, i = 1, 2
\]

\[
K = (C_y \cos \alpha - C_x \sin \alpha) / (C_y \sin \alpha + C_x \cos \alpha)
\]

Here \( C_p \) – pressure coefficient, \( S \) – body projection at the plane XOZ.

The problem is to find continuous functions \( f_1(x,z) \) and \( f_2(x,z) \), which minimize the functional (3) at given values of lift-drag ratio \( K \) (4), shape and area of the region \( S \), and body volume \( W \). This problem is equivalent to searching of maximum of the functional \( K \) (4) at given value of the integral (3).
Supposing that at the upper and lower surfaces of optimal body the Reynolds analogy is valid, the Stanton number can be replaced the friction coefficient \( k_i C_{fi} = S_{li} \), where \( k_i = \text{const} \).

Within the limits of thin bodies \( (u_i^2 << 1, \Delta_i = \Delta_{li}) \) and taking into consideration the closeness of \( k_i \) values (Fig. 3), the condition of prescribed heat flux (3) can be replaced by the condition of prescribed friction drag coefficient \( C_{sf} \)

\[
S_{b} C_{sf} = \int_{S} (C_{f1} \Delta_{11} + C_{f2} \Delta_{12}) dx dz
\]

(5)

Then the problem to find body shape, which minimize heat flux to the body side surface at given LDR value is reduced to the problem on optimization of the configuration of the body with maximal \( K_{\text{max}} \) at given value of friction drag coefficient \( C_{sf} \): to find in the class of continuous function the extremals \( y + f_i(x, z) = 0 \), \( y + f_2(x, z) = 0 \), which impart maximum to the functional (4) at given values of the integral (5), body volume, and shape and area of body projection of the plane \( XOZ \). As additional conditions there may by considered bluntness shapes of the nose and leading edges, base pressure coefficient, etc.

The problem formulated was solved with the help of the method of local variations [8]. The body surface was covered by a grid of small triangular meshes where values of pressure coefficient were determined with subsequent calculation of the integrals (1), (2) and lift-drag ratio (4). The optimization procedure consists in variation of the coordinates \( y_{ij} \) and \( y_{kl} \) of grid points on the body surface \( y_{ij} \pm \delta_{ij}, y_{kl} \pm \delta_{kl} \) and choice of variations \( (\delta_{ij}, \delta_{kl}) \), which increase \( K \) value keeping the body volume. Pressure coefficient is calculated through the tangent wedge formulas [4, 9].

\[
C_{p} = 2 \cos^2 (\pi, \bar{v}) \left( \frac{\gamma + 1}{4} + \left[ \left( \frac{\gamma + 1}{4} \right)^2 + \frac{1}{A^2} \right]^{1/2} \right), \cos(\pi, \bar{v}) > 0
\]

\[
C_{p} = \frac{2}{\gamma(M^2 - 1)} \left[ \left( 1 - \frac{\gamma - 1}{2} \right)^{2\gamma} - 1 \right], \quad \cos(\pi, \bar{v}) \leq 0
\]

\[
C_{p} = -\frac{2}{\gamma(M^2 - 1)}, \quad 1 - \frac{\gamma - 1}{2} < 0, \quad \cos(\pi, \bar{v}) < 0
\]

where \( A = \sqrt{M^2 - 1} \cos(\pi, \bar{v}) \), \( \bar{v} = \{ \cos \alpha, \sin \alpha, 0 \} \)

\( M \) – oncoming flow Mach number, \( \gamma \) – specific heat ratio, \( \pi \) – vector of local internal normal to the body surface.

For numerical realization of the algorithm there were assigned values of friction coefficient \( C_{fi} \) and \( C_{f2} \) instead of \( C_{sf} \). And the values of the coefficients \( C_{fi}, C_{f2} \) were corrected in the course of computations in order to keep prescribed value of the integral (5).

Using calculation results on flow around the delta wing with flat windward surface (Fig. 1a) at \( M=15 \) there were found configurations for wings of maximal lift-drag ratio (Fig. 1b, 1c), wing of minimal drag (\( \alpha=0 \), Fig. 1d) at given value of friction drag coefficient \( (C_{sf}) \), and wing of minimal friction drag at \( \alpha=0 \) (Fig. 1e).

The wing geometrical parameters are presented in section 1. Values of friction drag coefficients \( C_{sf} \) for delta wing with flat windward surface (Fig. 1a) at \( M=15 \) are adduced in the Table below and according to this Table the value \( C_{sf} = 0.0065 \) was used in computations.
For each wing in Fig. 1 there are indicated values $K_{\text{max}}$ and $\alpha_*$ obtained with the Navier – Stokes equations. Values calculated by the tangent wedge method are indicated in parentheses.

Configuration of the body of maximal lift-drag ratio at $C_{xf} = \text{const}$ does not vary practically at variation of the values $C_{f1}$ and $C_{f2}$. Curvature of windward surface in the plane $x = \text{const}$ decreases as the value $C_{xf}$ decreases. Variation of $\alpha_*$ value causes transformation of shape of the optimal wing section (Fig. 1b, c), still $K_{\text{max}}$ value varies insignificantly. Note that $|K_{\text{max}}|$ values for positive and negative angles of attacks are close, and $K_{\text{max}}$ values for all examined wings are close too.

It follows from calculation results that maximal lift-drag ratio of thin blunted wings is defined mainly by planform and $C_{xf}$ value.

3. Lift-drag ratio of optimal bodies and heat flux to their surfaces

Conclusion about minimum of heat flux to the side surface was drawn within limits of the simplest model of interaction between a gas flow and body that represents satisfactory the real physical process for integral force characteristics ($C_x$, $C_y$, $K$, …) only. Now discuss correctness of this conclusion using more accurate mathematical model described in section 1. There were computed non-equilibrium air flows around five wings of triangular planform (Fig.1) at $v = 5 \text{ km/s}$, $Re_l = 10^6$ in the angle of angles of attack $\alpha$ from $0 \, 10 \, 14^\circ$. Bluntness radii of noses and leading edges for all examined configurations equal $r = 0.01 l$, where $l$ – root chord length, sweep angle of the leading edge $\chi = 70^\circ$, volume $W = 0.0195 l^3$.

Calculation results on heat flux $Q_1$ to the wing windward surface and total heat flux $Q_2$ are illustrated in Figs. 4, 5 as dependencies against $K$. The plots in Fig.4 show that at $K = \text{const} Q_1$ values are close for all examined wings. Minimal values $Q_2$ are realized at the wing (d), and maximal – at the wing (e). $Q_2$ values at the wings (a), (b), (c) are close to each other.

![Fig. 4. Heat flux $Q_1(K)$ to windward surface.](image-url)
Along with total heat flux its local values $q_w$ on the wing surface are of interest. In this connection Fig. 6 illustrates distribution of heat flux over the surfaces in cross sections $x = 0.58l$ and $x = l$ for the wing (b) at $\alpha=10^\circ$ and wing (d) at $\alpha=8^\circ$. Indicated values of the angle of attack $\alpha$ correspond to similar values of lift-drag ratio $K = 2.77$. The dependencies $q_w(z)$ show that specific heat flow rate in the front part of the wing (d) are significantly higher than for the wing (b) with total heat flux $Q_1$ being equal (see Fig. 4). It is noted that if $K(\alpha_1) = |K(\alpha_2)|$, where $\alpha_1>0$, $\alpha_2<0$, then heat fluxes to the windward surface of optimal body are equal: $Q_1(\alpha_1) = Q_1(\alpha_2)$.

On the whole it is noted that with similar values of lift-drag ratio $K$ the optimal blunted wings have similar values of heat fluxes. And at the windward surfaces these values coincide.
Conclusion

Calculations of non-equilibrium hypersonic flows around blunted delta wings has shown the validity of the Reynolds analogy; this allows simplifying the statement of variational problem on a shape of a body side surface realizing minimal heat flux at given value of let-drag ratio.

Within the class of thin in longitudinal direction bodies, this problem is equivalent to the problem on a shape of a wing with maximal lift-drag ratio at given value of drag coefficient conditioned by friction forces.

A shape of cross section of the optimal wing depends on given angle of attack $\alpha_*$, corresponding to $K_{\text{max}} = K(\alpha_*)$. If $\alpha_*$ increases, curvature of windward surface decreases, and the value $K_{\text{max}}$ varies insignificantly.

At given values of $K$ and $C_x$, the shape of the body of maximal lift-drag ratio realizes minimal heat loading of its windward surface.

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REFERENCES