EVOLUTION OF THE PULSED WAVE PACKET IN A SUPersonic BOUNDARY LAYER

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There are two approaches of an experimental study of the laminar-turbulent transition mechanisms in boundary layers. In the first place, it is the evolution of natural disturbances in the boundary layer. In this case, their integral characteristics are generally considered and the power spectra are analyzed. Since the nature of origin of the natural disturbances is random, it is impossible to determine the wave characteristics of pulsations and quantitatively compare with the results of the linear theory of hydrodynamic stability. Another approach is the experiment in controlled conditions when disturbances with known initial characteristics are introduced into stream. This gives to determine the wave characteristics of excited disturbances in the boundary layer and directly compare the results with theoretical calculations.

Small scales of compressible shear layers and high frequencies of the Tollmien-Schlichting waves overlay strict conditions to sizes both transducers and the source of controlled disturbances. High-frequency glow discharge is an effective way of introducing controlled pulsations into the supersonic flow. Results of experimental studies on linear and weakly nonlinear development of instability waves in the supersonic boundary layer confirm this [1, 2].

At high-frequency discharge the perturbations, limited in space but periodic in time (so-called wave trains), are excited in the boundary layer. The results of the wave trains evolution can be compared with theoretical calculations, but to find of them in nature is problematic. Wave packets, which are localized not only in space but also in time, are well known in nature.

Recent progress in the study of the late stages of the transition at low subsonic speeds are related to the technique of pulsed excitation coherent structures in the boundary layer, localized in space and time [3]. Coherent structures were generated by both the "blowing-suction" method and the vibrating surface. In [3] was shown that the transformation of the "structure" in a turbulent spot was related to the origin and development of high-frequency disturbances on the coherent structure, which occurs near the peak of the local velocity gradient $\partial u / \partial z$.

Streaky structures have the local velocity gradient in the longitudinal direction $\partial u / \partial x$, on which may also appear the secondary high-frequency disturbances. The occurrence of wave packets "forerunners" in a domain of fronts streaky structures and their development in the subsonic boundary layer was studied by experimentally in [4]. Experiments have shown that the "forerunners" are packets of waves Tollmien – Schlichting. The pressure gradient of the external flow directly affects the occurrence and development of the wave packets "forerunners".

The method of excitation of wave packets in a hypersonic boundary layer was used in the experiments [5, 6]. Development of wave packets downstream and their breakdown into turbulent spots on the walls of the nozzle of the wind tunnel have been studied in these works. Measurements were carried out in quiet flow at Mach number 6. The development of both naturally occurring and artificially generated glow discharge wave packets was studied. Frequency of artificial wave packets was 200 Hz. Experiments [5] have shown that under quiet flow the boundary layer on the walls of the nozzle in the measurements domain was laminar. The peak frequencies of these natural wave packets were in agreement with second-mode computations. It is known, the second mode in the hypersonic flow is the most unstable. Wave packets had the linear and the nonlinear development, and then they transformed into turbulent spots. Measurements of the spatial
development of controlled disturbances [6] showed that the wave packets in the transverse direction have large scales. The formation of low-frequency structure, which had positive values of pressure pulsations, was observed downstream along the line of symmetry of controlled disturbances.

Investigations of the wave packet evolution at supersonic speeds have not yet conducted. The goal of this work was to study the evolution of wave packets in a gradientless supersonic boundary layer of a flat plate generated by pulsed glow discharge.

**Experimental setup.** Experiments were made in low noise supersonic wind tunnel T-325 of ITAM SB RAS at Mach M = 2 and unit Reynolds number Re1 = 6·10⁶ m⁻¹. The model of a flat plate with a sharp leading edge, which was mounted at zero angle of attack, was used in the experiments.

Controlled disturbances in the boundary layer were introduced by surface glow discharge, consisting of two copper electrodes separated by a layer of dielectric. Electrodes were placed along the direction of the incoming flow. The minimum distance between the electrodes was 0.9 mm. High-voltage pulses from the ignition scheme of the glow discharge fed to electrode the nearest to the model leading edge. This electrode located at a distance x = 26.6 mm. A source of controlled disturbances is shown schematically in Fig. 1.

![Fig. 1. Source of controlled disturbances on a flat plate: view from top - on the left; (b) – side view - on the right; 1 – electrodes, 2 – dielectric.](image)

The basis for the discharge ignition circuit was a schematic circuit of automotive ignition. Spark coil with an inductance of 0.3 mH primary winding and secondary winding – 500 mH was used in the experiments. The scheme of the discharge ignition is based on a breakdown of the primary circuit of a spark coil. The breakdown of the primary winding ignition coil is carried out by a powerful bipolar transistor operating in switching mode. A control signal of square shape with the constant duration T = 0.6 ms was fed to the transistor. Frequency of the control signal was 200 Hz. Variable resistance R' in the primary winding of the ignition coil is used to change the current flowing in the primary circuit, and, accordingly, adjusting the power supplied to the discharge. A time of the burning of the glow discharge in supersonic flow was about 25-30 μs.

The disturbances measurements were conducted by a constant-temperature hot-wire anemometer (CTA) in supersonic boundary layer. Single tungsten wire in diameter 10 μm and length 1.5 mm was used as probe. The wire probe overheating was equal to 0.8, and the measured disturbances by CTA mainly consisted of mass flow pulsations. An automated measuring system was used in the experiments. It consists from equipment in CAMAC standard with CC-32 fast controller and PC computer. AC and DC signal from the hot-wire anemometer were written in computer using of 12-bit analog-to-digital converter (ADC) with sampling rate 1.25 MHz and by DC digital voltmeter Agilent 34401A. Time traces in length of 4096 points ADC were synchronized with glow discharge. To increase the signal/noise ratio synchronous signal summing of 320 realizations was carried out.
Results. Waveforms of pulsations measured at the same point \((x = 60 \text{ mm}, z = 0 \text{ mm}, y = 0.58 \text{ mm})\) are represented in Fig. 2. Under the conditions of the discharge ignition, the pulsating signal from anemometer consists of disturbances from the glow discharge and naturally occurring fluctuations in the boundary layer. This can be seen in Fig. 2, \(a\). Result synchronous averaging of the 320 measured waveforms is shown in Fig. 2, \(b\). The method of synchronous averaging is a good filter and outside of borders of the artificial wave packet the pulsations amplitude is close to zero. As can be seen from Fig. 2, \(a\), it is possible to select an area where the present together natural and controlled pulsations and area, where attend only natural disturbances. Spatial development of these natural disturbances in a supersonic boundary layer was identical to the results of experiments without discharge. Therefore, it seems possible to study the evolution of both the controlled and natural disturbances in a supersonic boundary layer under conditions of controlled experiment. Let's introduce the following notation: \(m'_{\text{total}}\) – total pulsations (natural + artificial); \(m'_{\text{nat}}\) – natural disturbances; \(m'_{w.p.}\) – wave packet generated by the discharge.

Profiles of mean flow and the root mean square total pulsations along coordinate \(y\) are shown in Fig. 3. The values of mean flow, normalized to the level of mass flow in the external flow, are estimated by a simple dependence for supersonic flow: \(E^2 \approx K \cdot (\rho U)^{2.5} \cdot x\). Root mean square (RMS) magnitudes of total pulsations were determined over the entire length of measured waveform (4096 ADC samples). The measurements of profiles were performed at \(z = 0 \text{ mm}\), along the central line. The thickness of boundary layer increases downstream and experimental estimates of the thickness coincide with the theoretical dependence: \(\delta = \eta x / \sqrt{\text{Re}_x}\). Maximum of RMS fluctuations is located in a supersonic part of the boundary layer. It practically doesn't increase in amplitude downstream. It is observed the expansion of the maximum in boundary layer thickness.

The pulsations of mass flow and the mean flow are components of a signal output from hotwire anemometer. These values are measured simultaneously at each point in space. Therefore, for the profiles along \(y\) coordinate, it can plot dependences of the RMS pulsations from mass flow values normalized to the external flow. On the data presented in Fig. 3, these dependences were plotted in Fig. 4. Curve 1 corresponds to the natural pulsations \((m'_{\text{nat}})\); curve 2 – total signal \((m'_{\text{total}})\) and the data on controlled disturbances \((m'_{w.p.})\) are shown on curve 3. RMS of total signal and the wave packet were determined by the entire length of measured waveforms (4096 ADC samples). RMS values for natural disturbances were calculated from the right half of measured waveforms (2048 ADC samples).
In the first measured profile, $x = 60$ mm, natural and total pulsations have similar values. Maximum of pulsations in both cases coincide and is located in the area $(\rho U)/(\rho U)_\infty \approx 0.6 \div 0.8$. RMS values of corresponding controlled pulsations are several times less than the level of natural disturbances.

The largest differences in the evolution of natural and controlled disturbances are observed at $x = 100$ mm. Total pulsations exceed natural level on the entire thickness of the boundary layer. Controlled disturbances, curve 3, have two maxima, which are located in the areas above and below the layer of maximum pulsations.
Distributions of RMS fluctuations of mass flow downstream were shown in Fig. 5. Here, as in Fig. 4, the natural pulsations are given in the curve 1; full pulsations – 2; controlled perturbations are plotted in the curve 3. The data were obtained in the layer of maximum pulsations. Measurements were performed at \(\rho U \approx \text{const}\). Amplitude of controlled disturbances near discharge \((x = 40\text{–}60\ mm)\) is small and their contribution to the process of transition to turbulence is negligible. But, beginning from \(x > 80\ mm\), the RMS level of controlled pulsations increases considerably. Increments of growth for total signal are higher than in the case of natural disturbances. Therefore the pulsed discharge leads to some distortion of the boundary layer and shifts the beginning of the laminar-turbulent transition upstream.

![Fig. 5. Growth curves of integral pulsations downstream.](image)

\(I – \text{natural pulsations, 2 – total signal, 3 – artificial disturbances}\)

Strong growth of controlled disturbances and total pulsations can probably be explained only by the nonlinear interaction of the wave packet and natural fluctuations in the boundary layer. Furthermore, the wave packet has a continuous spectrum in the frequency domain and capabilities appear for occurrence of a subharmonic resonance. Note that for the subharmonic resonance the pulsations become related to both over frequencies and over the wave vectors.

It was mentioned above that the signal measured by hot-wire anemometer consists of natural and controlled pulsations. Then,

\[
\langle m'_\text{total} \rangle^2 = \frac{1}{N} \sum (m'_\text{total})^2 = \frac{1}{N} \sum (m'_{\text{nat}} + m'_\text{w.p.})^2.
\]

Removing the brackets we obtain:

\[
\langle m'_\text{total} \rangle^2 = \langle m'_{\text{nat}} \rangle^2 + \langle m'_\text{w.p.} \rangle^2 + \frac{2}{N} \sum (m'_{\text{nat}} \cdot m'_\text{w.p.}).
\]

Let’s express the last term of this equation and normalize on the level of natural pulsations, and resulting value let’s denote as \(\Delta\):

\[
\Delta = \frac{\langle m'_\text{total} \rangle^2 - \langle m'_{\text{nat}} \rangle^2 - \langle m'_\text{w.p.} \rangle^2}{\langle m'_{\text{nat}} \rangle^2} \cdot 100\%.
\]

If the wave packet and naturally occurring disturbances don’t interact with each other (i.e., these are independent), then the value of \(\Delta\) will tend to zero. In the case of the nonlinear interaction the natural and artificial disturbances will be linked, and the value of \(\Delta\) will have nonzero values.

Variations of \(\Delta\) downstream were shown in Fig. 6. Up to \(x = 70\text{–}80\ mm\) the level of \(\Delta\) is within 2.5\% modulo. Significant growth of \(\Delta\) is observed at \(x > 80\ mm\), which indicates the relation of the
wave packet with the pulsations of natural origin. That is, the nonlinear interaction of natural and controlled disturbances probably occurs at \( x > 90 \) mm.

Evolution of the artificial wave packet downstream was shown in Fig. 7. Wave packet has clear bounds in time. It can be estimate the propagation velocities of the wave packet downstream by the position of its bounds in the plane \((x, t)\). Estimates show that the leading edge of the wave packet propagates with a velocity about 0.9 \( U_\infty \), the speed of the trailing edge is about 0.3 \( U_\infty \), center of the wave packet – approximately 0.6 \( U_\infty \). Note that these estimates are close to the results obtained in [7]. Variation of the wave packet amplitude several times, apparently does not affect its propagation velocities downstream.

![Fig. 7. Evolution of waveforms downstream at \( z = 0 \) mm (a) and corresponding to them amplitude spectra (b).](image)

Amplitude-frequency spectra for waveforms shown in Fig. 7 (a) are represented in Fig. 7 (b). Near the discharge, \( x = 40 \) mm, the spectral composition of the wave packet is bounded above by frequency 40 kHz. Up to \( x = 75 \) mm only low-frequency disturbances are amplified. At \( f > 5 \) kHz amplitude spectra lie on the same curve. Further, up to \( x = 107 \) mm, there is a decrease of spectral harmonics of the wave packet in the frequency range \( f = 5–40 \) kHz. The low-frequency part of the spectrum continues to increase. At \( x > 107 \) mm only the low-frequency perturbations amplify, in the frequency range \( f = 5–40 \) kHz the curves coincide and disturbances close to neutral.

Contour lines mass flow pulsations of the wave packet in the plane \((z, t)\) at \( x = 60, 80, 100 \) mm were plotted in Fig. 8. Corresponding contour lines of amplitude spectra in the plane \((\beta, t)\) were shown in Fig. 8. On the graphs, positive values of pulsations are shown by solid lines, negative values – dashed. From the data presented in Fig. 8 it is seen that the wave packet is expanded in the spanwise direction, which is spread downstream. The wave packet shape does not undergo significant changes. On contour lines in the plane \((z, t)\) the estimates of a spreading of the wave packet were made. Half-angle spanwise spreading of the wave packet is about 5°, which is close to the results of experiments of a linear development of controlled disturbances with high-frequency discharge [1]. It should be noted that the result is different from the estimate made in [7], where the single wave packets of large-amplitude were introduced into supersonic boundary layer, whose scales are comparable to the size of the experimental model.
Fig. 8. Contour lines of the wave packet in the plane \((z, t)\) at \(x = 60, 80\) and \(100\) mm – (a, c, e) and corresponding to them contour lines of amplitude spectra in the plane \((\beta, t)\) – (b, d, f).
At $x = 60$ mm (see Fig. 8,a), areas of the local flow defect in time clearly are outlined by lines of equal amplitude. Positive pulsations are at the stage of its formation. Negative values of pulsations are focused in the areas at $z = 0, \pm 2$ mm. The low-frequency structure with positive values is formed at $z \approx \pm 1$ mm. The spatial evolution of the wave packet (see Fig. 8,e), leads to an increase its amplitude and spanwise expansion of the local flow defect in time. But the "positive" structure doesn't practically change its shape downstream.

The results of spectral analysis on the transverse wave numbers were presented in Fig. 8,b, d, f. Since the data in planes ($z, t$) and ($\beta, t$) are shown in Fig. 8, it is possible to compare the structure of controlled disturbances in physical and wave space. At $x = 60$ mm (Fig. 8,b), «instantaneous" flow defect, which forms the leading edge of the wave packet, in the wave space is displayed in kind of three areas at $\beta = 0, \pm 3$ rad/mm. Formation of the "positive" structure occurs slightly later on time, at $\beta \approx \pm 1$ rad/mm. Thus the wave packet at the initial stage of its development consists of two-dimensional and three-dimensional waves.

Further downstream, amplitude peaks almost do not change its position on the transverse wave numbers. There is a shift of the wave packet in time. The low-frequency "positive" structure in wave space represents itself the two extremes at $\beta \approx \pm 1$ rad/mm, which converge to each other and they are combined into a single structure in area of the trailing edge of the wave packet (Fig. 8,d, f). The longitudinal dimension of this structure significantly rises downstream.

If look at the results of experiments in an incompressible boundary layer [3, 8], where was studied the evolution of artificial streaky structures generated by pulsed blowing through the gap, can see some similarities with data presented in Fig. 8. To explain of these some similarities of the development of localized structures in the subsonic and supersonic boundary layer, probably, may as follows. Pulsed glow discharge affects the supersonic boundary layer two ways. First of all it is the thermal effect, because the temperature of the discharge plasma can reach several thousand Kelvin. On the other hand, the low temperature plasma plays the role of obstacle, thereby locally displacing shear layers to the upper edge of the boundary layer. Local blowing similarly affects the subsonic boundary layer.

**Conclusion.** The experimental study of the development of the single wave packets in a supersonic boundary layer on a flat plate at $M = 2$ was made. Wave packets were excited by short ($\sim 25$ mks) pulsed glow discharge.

Wave packet has a continuous spectrum, bounded above the frequency $f=40$ kHz. Its evolution downstream leads to increase of low-frequency component of the amplitude spectrum. The wave packet in the initial stage and natural perturbations develop independently of each other. Further downstream, the wave packet and natural pulsations become related, which may indicate about their nonlinear interaction.

Propagation velocity of the leading edge of the wave packet close to the free stream ($0.9U_\infty$), the speed of the trailing edge of the wave packet is subsonic ($0.3U_\infty$). These estimates of the wave packet propagation coincide with the development of large-scale structures in a supersonic flow in case a pulsed glow discharge of long duration. Half-angle spreading of the wave packet in spanwise direction is approximately equal to $5^\circ$, which is close to the results of studies of the linear development of wave trains in supersonic boundary layer at $M = 2$. The wave packet near the source of controlled pulsations develops inside the boundary layer. However the evolution of controlled disturbances leads to their radiation into free flow.

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