RELAXATION FEATURES OF A TURBULENT BOUNDARY LAYER IN AN UNFAVORABLE PRESSURE GRADIENT

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1. Introduction

The relaxation of a shear flow (the process of an equilibrium state recovery) caused by the presence of a disturbance source such as a wall fence, roughness elements or obstacle in a turbulent boundary layer is a slow process which can not be predicted, at least accurately, even on the basis of current turbulence models. Previous investigations [1 – 3] are devoted to the study of the process of the boundary-layer relaxation under the conditions of gradient-free streamwise flow. However, the majority of practical cases which deal, for example, with the development of optimum design of heat exchanger channels, of elements and units of airspace technology, is related to the class of nonequilibrium (by Clauser) flows forming in unfavorable pressure gradient. To the authors knowledge, paper [4] is the only which is actually devoted to the study of the turbulent boundary layer relaxation under the conditions of unfavorable pressure gradient for the case of flow around a obstacle of height $h$. Nearby the equilibrium state has not been reached right up to the last measuring cross section. It is quite evident that the studying in the conditions mentioned above of the behavior of a nonequilibrium turbulent boundary layer disturbed by the presence of a source of any geometry, is more actual problem.

2. Experimental conditions and techniques

The experiments were carried out in a subsonic wind tunnel T−324 of the ITAM SB RAS with the freestream flow velocity in a reference cross section of $U_\infty = 25.0$ m/s which corresponded to a unit Reynolds number of $Re_1 \approx 1.66 \times 10^6$ m$^{-1}$.

The measurements were performed on a model of flat plate 1 made of D16T alloy with the sizes of 2500×993 mm$^2$ in a plan, and of 6 mm thick. It was mounted horizontally in the test section of the wind tunnel on guiding rails 2 (Fig. 1). Both leading and trailing parts of the plate, as viewed from the non-operating side, are of semi-elliptical form with half-axes ratio of $b/a = 1:12$. The form of the proper leading edge is characterized by a radius rounding $r = 0.4$ mm. On the symmetry axis (of the flat plate) a group of static pressure taps of 0.4 mm diameter is situated. Their coordinates are presented in the Table.

![Fig.1. Simplified layout of the experiment](image)

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paper 6 (with smoothed steps) of 15 mm length and 0.6 mm height which was glued over the plate span in the region of the maximum change in pressure observed in the vicinity of the leading edge.

Transversally flown circular cylinder 10 in a steel polished rod diameter 2 mm, was used as a source of disturbances. It was situated directly on the surface in the developed turbulent boundary layer at the distance of 590 mm from the plate leading edge. The relative diameter of the cylinder \(D/\delta_0\) was of 0.117 mm where \(\delta_0\) is boundary layer thickness in the point of cylinder position. This value equal to 17 mm is defined under the condition that \(U/U_e = 0.99\).

To generate a boundary-layer flow with a streamwise gradient, special adjustable dummy wall 9 was used. It was made of a sheet of glass-cloth laminate of 3.7 m length and 2.5 mm thick which was fastened to the wind-tunnel ceiling with cross-mounted guides allowing to vary a wall profile along the flow.

To carry out the measurements in the boundary layer, a remote-control traversing gear 7 was used. Together with the standard traversing gear of the wind tunnel, it ensured an accuracy of \(\pm 0.01\) mm in the \(y\) direction and \(\pm 0.5\) mm in the \(x\) and \(z\) directions.

The mean velocity in the examined point of the flow field and the streamwise component of velocity fluctuations were measured by DANTEC hot-wire equipment. The primary transducer was a miniature hot-wire probe with a single straight sensor made as a tungsten wire of 5 \(\mu\)m in diameter and 1.2 mm active long.

To determine local values of a skin friction coefficient a Preston method was applied. Depending on test conditions, three or two tubes were used; their outer diameters were 1.06, 1.61, and 2.01 mm, and the ratio of the inner to the outer diameter was 0.62. Computation of the skin friction coefficient \(C_f\) was carried out with the use of a modified Patel’s calibration dependence as a averaged value from the results of measurement with several tubes.

### 3. Results of the experiments

**3.1. Characteristics of the initial flow.** In the course of experiments an unfavorable pressure gradient was organized along the surface, which dimensionless parameters \(\beta\) and \(P^+\) varied streamwise insignificantly and in the region of the plate trailing edge they reached the values of 2.18 and 3.86 \(\cdot 10^{-3}\) correspondingly, where \(\beta = (\partial^x/\tau_w) dP/dx\), and \(P^+ = \nu (dP/dx)/\rho v e^{-3}\).

Under these conditions the experimental profile of the mean velocity is not separated anywhere. At the measured values of \(\bar{\tau}\) the obtained distributions of \(U/U_e = f(y)\) contain nothing unusual and correlate entirely with a canonical profile recommended by the Proceedings of the known Stanford conference of 1968. The presence of a developed wake region the role of which increases with increasing coordinate \(\bar{x}\) is a typical phenomenon for flows with an adverse pressure gradient, which is well known in the literature.

This feature is additionally confirmed by the profiles of the streamwise component of velocity fluctuations in the form \(\sqrt{\bar{u}^2}/U_e = f (\log y^+ )\). They exhibit a typical (as compared to the case of a gradient-free flow) deflection of the velocity turbulent fluctuations towards great \(y\) values. This deflection increases as the distance \(x\) of the model leading edge rises, which is quite easily explained. At the same time, the near-wall maximum of the fluctuations \(\sqrt{\bar{u}^2}/v_*\) is situated at the distance of \(y^+\approx13\), which is, generally speaking, close to the observed value in the gradient-free flow.
Fig. 2 shows the data of wall shear stress distribution over the plate with disturbance source being absent, as a dependence of $\tau_w = f(\Delta x/D)$ (open circles). ($\Delta x/D$ is a conventional coordinate used because the data obtained with the disturbance source are also given here – see below). As evident, here takes place the usual course of the dependence of $\tau_w = f(\Delta x/D)$ which occurs in similar situations with Re variation, namely, the significant decrease of a skin friction over the model length which is typical in the presence of the adverse streamwise pressure gradient.

It is known that Clauser’s nonequilibrium parameter $G$ [5] serves as a criterion of estimation of a shear flow relaxation to the equilibrium state:

$$G = \frac{2}{C_f} \left( \frac{H - 1}{H} \right),$$

where $C_f$ is a local skin friction coefficient. In the presence of the streamwise pressure gradient, Nash [6] has offered the following dependence

$$G = 6.1(\beta + 1.81)^{0.5} - 1.7,$$

which is valid also in a special case of $\beta = 0$.

In Fig. 3 in generalized coordinates $F_f - F_p$ [7] where $F_f = 1/G^2$ and $F_p = \beta/G^2$, one can see the data of variation of Clauser’s nonequilibrium parameter over the length of the plate when the disturbance source is absent in the boundary layer. The meaning of application of such coordinates is as follows. In the gradient flows (in $dC_f/dx > 0$) $C_f \to 0$. Then the value of $G \to \infty$, and the defining parameter $G$ is connected with a high error. It is clear that the application of the generalized coordinates is more preferable.

Fig. 2. Wall shear stress distribution of over the model length: open circles – without cylinder; closed circles – with cylinder.

Fig. 3. Illustration of the equilibrium stage of a flow without cylinder in generalized coordinates. Experiment: Bradshow (1); East and co-authors (2); Good and Joubert (3); Nigim and Cockrell [9] (4); present paper (5); by East and co-authors formula (6). (All references to the authors in [9]).
In such a view the data obtained in the present work (the open circles) agree successfully with the experimental results of the other authors and may be generalized with the satisfactory accuracy by an empiric dependence

\[ F_f = 0.024 - 0.8 F_p, \]

recommended by East and co-authors [9]. Hence it also follows that the equilibrium character of the flow in the boundary layer under consideration begins at the distance of about 600 – 700 mm from the model leading edge.

3.2. Flow properties with the disturbance source. As in the previous case, the experimental velocity profiles in the variables of the law-of-the-wall \( U' = f(\log y^+) \), with various values of \( \Delta x/D \), generally successfully correlate with the canonical profile. In the initial cross sections (see, \( \Delta x/D = 15 \)) where the role of the disturbance source is significant, a comparatively narrow logarithmic region and a developed wake region are formed over the boundary layer height. As moving far downstream of the cylinder, the velocity profile changes fast which becomes apparent in expanding logarithmic region and weakening wake region. Further increase of the streamwise coordinate is characterized by a new development of the wake region caused by the effect of the unfavorable pressure gradient along the surface.

The profiles of the streamwise component of the velocity fluctuations \( \sqrt{u'^2} / U_c = f(\log y^+) \) reveal the significant amplification of the turbulent velocity fluctuations which are practically twice larger in the near flow region (\( \Delta x/D = 5 \)) than the corresponding values in a “undisturbed” flow at the same values of the streamwise coordinate \( \bar{x} \). Moving downstream of the disturbance source, the profiles \( \sqrt{u'^2} / U_c = f(\log y^+) \) change very fast in this region. Later downstream, however, the relaxation rate slows down, and the character of the fluctuation profiles near smoothly towards the corresponding classical distribution.

As for the distribution of integral characteristics, it turned out that directly near the cylinder the values of displacement thickness \( \delta^* \) and moment thickness \( \delta^{**} \) are considerably higher as compared to the corresponding values when the disturbance source is absent. But a certain decrease of \( \delta^* \) can be observed in the region of the flow of \( 5 \leq \Delta x/D \leq 50 \). It is caused by mass flow rate re-distribution in the region pointed. With the streamwise coordinate further increase the growth rate of the integral characteristics increases, and as a result, even at \( \Delta x/D \approx 400 \) they slightly exceed the corresponding values in the “undisturbed” flow.

It is interesting that the distribution of the shear stress \( \tau_w = f(\Delta x/D) \) over the plate (see Fig. 2, closed circles) is not so conservative to the disturbance influence as compared with the integral values \( \delta^* \) and \( \delta^{**} \). Right behind the cylinder (\( \Delta x/D \approx 5 \)), i.e. in the zone of a

![Fig. 4. Variation of Clauser’s nonequilibrium parameter over the plate length with cylinder: 1 – present paper, 2 – Nash formula [6].](image-url)
recirculating flow, \( \tau_w \) is several times lower than the corresponding value typical for the equilibrium state (open circles). As \( \Delta x/D \) rises, \( \tau_w \) increases dramatically approaching to its value at the equilibrium flow. The total achievement of the equilibrium state by the shear stress occurs at a distance of \( \Delta x \) from a cylinder axis of about 170D. Thus, the rate of this value recovery is very high but this fact cannot be a basis for the spreading over the whole average flow.

In Fig. 4 the dependence of \( G = f(\Delta x/D) \) presents the data of Clauser’s nonequilibrium parameter over the plate length (circles). Here also are presented the results obtained on the base of Nash dependence for the equilibrium gradient flow (curve). One may note that the process of flow recovery to the equilibrium state occurs in two stages. In fact, first the process progresses very intensively, with the result that the intermediate equilibrium state is achieved already at \( \Delta x/D \approx 170 \). With the further downstream motion of the disturbance source, however, the curve of \( G = f(\Delta x/D) \) crosses the equilibrium position, value \( G \) reaches its maximum, and the recovery process becomes sluggish. Within the limits of the measurement error, which doubled value of \( \pm 2\sigma \) is indicated with a vertical line, reaching of the equilibrium state takes place at a distance of about 420D from the cylinder axis. It exceeds considerably the corresponding value obtained in a gradient-free flow which is of about 230D, all other factors being equal \[10\].

The profiles of the streamwise velocity component, both with the cylinder being absent on the surface (in the undisturbed boundary layer), and at cylinder being present, served as a basis for the estimation of the length of a nonequilibrium flow caused by the relaxation of the turbulence parameters. For this purpose an approach was used which had been applied in \[11\], where an attempt had been made to perform this estimation on the base of a non-dimensionalized thickness of the excess kinetic energy for the fluctuating velocity as follows

\[
E_t = \left[ (u^{-eq} - u^{-}) \right] d(\frac{y}{\delta}) = \frac{1}{\nu} \int_0^{\frac{\delta}{y}} \left[ u^{-eq} - u^{-} \right] d(\frac{y}{\delta}),
\]

where index “eq” relates to the equilibrium state conditions.

The results obtained are presented in Fig. 5 as a dependence of \( -E_t = f(\Delta x/D) \). It is evident that in the initial stage (before \( \Delta x/D \approx 200 \)) the relaxation process occurs very intensively which results in 75 % recovery. Downstream the recovery process by its character resembles the asymptotic one. The equilibrium state of the boundary layer is reached at a distance of about 750D from the cylinder axis. It means that the turbulent characteristics of the boundary layer are more sensitive to the influence of two-dimensional disturbances than the mean flow parameters. Thus, the memory of fluctuating field to the two-dimensional disturbances is obviously longer than the similar value for the mean flow.

![Fig. 5. Variation of non-dimensionalized thickness of the excess kinetic energy for the fluctuating velocity over model length.](image)
4. Conclusion

(i) The distribution of the mean and fluctuating parameters of the flow over the model length indicates the formation of the two stages of the relaxing process – a fast one which covers the flow region from 120 to 200D, and the following slower one which distributes up to the end of the nonequilibrium region.

(ii) The process of recovery of the shear flow characteristics is unequal. Specifically, the shear stress returns to the equilibrium value at a distance of $\Delta x \approx 170D$ from the cylinder axis, which indicates the fast rate of the relaxation of this quantity.

(iii) The distribution of Clauser’s nonequilibrium parameter over the model length shows that the equilibrium state of the boundary layer is reached at a distance of $\Delta x$ of about 420D. It is much higher than the corresponding value obtained in the gradient-free flow around the flat plate, all other conditions being equal.

(iv) The characteristics of the near-wall turbulence are more sensitive to the disturbing influences as compared to the mean flow parameters. In this case the equilibrium state of the boundary layer is reached at a distance of $\Delta x$ of about 750D. Thus, the turbulence memory to the two-dimensional disturbances is obviously longer than the similar value for the mean motion.

REFERENCE