Introduction

The investigation of a supersonic flow around a wing at the high angle of attack is of practical interest. When a delta wing is flowed around, a complicated pattern of a flow on the leeward side can be observed. Here streamwise vortex structures, “hanging” shock waves, rarefaction waves form. The flow structure on the delta wing leeward side can also be unsteady. A great body of experimental and theoretical works is devoted to the topology of the flow on the wing leeward side. However, the existing classifications of modes require further specification and improvement. Therefore, new experimental research will be carried out with the advent of new measurement techniques. Numerical research might also make their contribution to the study of the mechanism of this interesting phenomenon and classification improvement. It should be understood how much would the existing models of gas dynamics be suitable for the simulation of the flow on the leeward side of the wings, i.e., how much adequate to the physical reality is it.

Obtaining numerical solution of complete Navier–Stokes equations is still a labor-consuming process which is not always rewarded with good results. There is quantity of reasons explaining this situation. The main one is likely to be the problem of closure of Navier–Stokes equations.

The model of boundary layer being used, incorrect actions such as attempts to study the separation are eventual which conflicts with the initial boundary layer model [1].

The classic Prandtl model also has its disadvantages (a non-viscous flow + a boundary layer) [1, 2].

The issue of possible simulation of these processes within Euler equations limits reminds still open. On the one hand, the many of methods of Euler equation solution presented in literature possess high accuracy and are physically proved. They allow to obtain many important characteristics of a flow, since though no real flow is completely non-viscous, the effect of viscosity is essential only in narrow regions such as boundary layers. On the other hand, it is opined that vortexes on the wing leeward side at Euler equation numerical solution can be obtained only in presence of a sharp edge, the reason of vortex appearance being resulted from numerical dissipation [3–5].

Actually, the numerical dissipation can be not the only one vortex source in the non-viscous flow. The present paper shows that the assignment of a vortex surface instead of ordinary slip condition on a rigid wall permits to obtain the numerical results which describe adequately the physical processes within the limits of Euler equations.

In fact, the solution of a gas-dynamic task for Euler equations will be correct if the succession with more complete approximation (Navier–Stokes equations) is applied when a small parameter (viscosity) tends to zero [1, 6, 7]. It is possible if the functions determined by the system of gas-dynamic equations belong to the class of bounded functions (BV) [8, 9]. It follows from the function boundedness requirement that the gas-dynamic parameters determined by the equations can be presented as the expansions of integral or fractional power of normal coordinate to the wall. It corresponds to the description of various gas-dynamic peculiarities [10]. In the case of fractional power expansion (Golubinskii peculiarity) [10, 11] the solution

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agrees with the wall law in a turbulent flow with disappearing viscosity [12 – 14]. The solid surface here is a vortex surface, or rather a surface with vortices lying on it.

The sources of vorticity appearance at Euler equation numerical solution

At the numerical solution of Euler equations the vortex structures were observed in some instances [3, 4, 16 – 22]. The current opinion is that the reason of vortex forming is viscosity and, in particular, numerical dissipation at the numerical solution of Euler equations. Actually, viscosity is just one of possible reasons of vorticity appearance in a flow. Three sources of vortex forming can be named at the numerical solution of Euler equations.

1) Flow nonuniformity by entropy or full enthalpy:

It is known that curvilinear shock waves can generate the vortex structures [16]. Use of approximated nonuniformity assignment [17] permits to obtain the pattern of large-scale separated flow within the framework of a non-viscous gas model.

2) Numerical dissipation:

Research of [19, 20] have demonstrated that vortex flows similar to those occurring at Navier–Stokes equation solutions appear at quite rough computational grid for the non-viscous gas model. These effects, however, can be unobserved on small cells of the computational grid. Indeed, Helmholtz theorem takes place for the Euler equations in the cases of barotropic liquid or isentropic flow, i.e., from the equation it follows that [19]

\[
\text{helm } \Omega = \text{rot} \left( \frac{d\sigma}{dt} \right) = -\rho^2 \text{grad} \times \rho \text{grad} \ p = 0 .
\]  

Here \( \rho \) is density, \( p \) is pressure, \( t \) is time. The right part in equation (1) can differ from zero due to the errors in approximation and by that generate the vorticity in the flow.

3) Surface of the body flowed around if it is defined as a vortex surface:

Quite successful results of the vortex flow numerical simulation are reported [21, 22] for subsonic flow when the flowed surface is alternated by a vortex system (discrete vortex method). Let us consider the possibility of the vortex surface defined at the supersonic flow.

Let us analyse the behavior of Euler’s equation solution near a wall in the context of bounded functions (BV).

\[
\frac{\partial g}{\partial t} + \vec{A}(g) \ \text{grad}(g) + H(g) = 0 ,
\]  

where \( \vec{w} = w (u, v, w) \) is the velocity vector, \( \vec{\xi}_1, \vec{\xi}_2, \vec{\xi}_3 \) are orts of the natural coordinate system \( \vec{\xi}, \eta, \zeta \) correspondingly. Axis \( \xi \) is directed by the normal towards the surface, \( \vec{\eta}, \vec{\zeta} \) lie of the body surface. Coordinate lines \( \xi \) and \( \eta \) have the radius of curvature of \( R_1, R_2 \).
Let us deal with the possible view of the peculiarity by \( \zeta \). The sought function \( g \) from (3) can be presented as a product

\[
g(\xi, \eta, \zeta) = \Psi(\xi, \eta, \zeta) \cdot \varphi(\xi, \eta, \zeta, t)
\]

where function \( \Psi(\xi, \eta, \zeta) \) is independent on time includes maximum possible peculiarity by \( \zeta \) and has no peculiarities by \( \xi \) and \( \eta \), \( \varphi(\xi, \eta, \zeta, t) \) – a certain function with no peculiarities. Then, (2) can be re-written as

\[
\psi \frac{\partial \psi}{\partial t} + \mathbf{A}(\varphi) \psi (\nabla \varphi \cdot \psi + \psi \cdot \nabla \varphi) + \psi^2 H(\varphi) = 0 .
\]

(4)

From the requirement of function \( g < M < \infty \) boundedness it follows that \( \Psi < M_1 < \infty \).

Let there near the wall be \( \lim_{\zeta \to 0} \frac{\partial \psi}{\partial \zeta} = \infty \). From (4) it follows that

\[
\lim_{\zeta \to 0} \Psi \left( \frac{\partial \psi}{\partial \zeta} \right) = C(\xi, \eta) < \infty .
\]

(5)

Then from (5) and the requirement of function boundedness it follows that \( \lim_{\zeta \to 0} \psi = 0 \) and integration (5) given the solution in the form of

\[
\psi = C_1(\xi, \eta) \zeta^{1/2} .
\]

(6)

Thus, function \( g \) (3) can be not only regular (\( g_{\text{reg}} \)) but also can have the peculiarity \( \zeta^{1/2} \) \( (g_{\text{irreg}}) \), then, it can be presented as

\[
g = g_{\text{reg}} + g_{\text{irreg}} = \sum_{k=0}^{\infty} g_{k/2} \zeta^{k/2}
\]

(7)

where \( g_{k/2}(\xi, \eta) \) are the expansion coefficient. The kind of these coefficients was determined in [23]. There was analyze of different cases these coefficients. In [18] it was shown that at \( g_{(2k-1)/2}(\xi, \eta) \neq 0 \) and \( \xi \to 0 \) the module of vortex \( \Omega \) vector has the value of \( |\Omega| = O(1/\zeta^{1/2}) \), and the normal component of vortex \( \Omega z = O(1) \). Consequently, the angle between the vortex direction and normal at \( \zeta \to 0 \)

\[
\cos \gamma = \Omega z / |\Omega| \to 0 .
\]

(8)

It means that the vortex lies on the wall. I.e., the gas-dynamic parameters being decomposed by the fractional degrees, the wall is the vortex surface.

In [23] it was shown that coefficients \( u_0 \) and \( v_0 \) (at the wall) can be assigned arbitrary. Let them be defined with the requirement of agreement between Euler equation solution and more full approximation solution which is Navier – Stokes equations in respect to Euler equations, the viscosity tending to zero. Let us use the technique of Vishic and Lusternic asymptotic expansion joining [24].

Let us consider Navier – Stokes equation in the form of

\[
\frac{d \mathbf{\Pi}}{dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{\Pi} + \frac{v}{3} \nabla (\text{div} \mathbf{\Pi}) .
\]

(9)

Here \( v \) is a coefficient of kinematic viscosity, \( \nabla^2 \) is Laplacian.

For the constant flow it can be written in a dimensionless form
\[
(\nabla \nabla) \overline{w} = -\frac{1}{\rho} \nabla p + \frac{\nu}{\text{Re}} \nabla^2 \overline{w} + \left(\frac{4}{3} \frac{\nu}{\text{Re}}\right) \nabla(\nabla \overline{w})
\]  

(10)

where \(\nabla\) is inverted delta, \(\text{Re}\) is Reynolds number.

We shall deal with the solution of equation (1) near the wall (when \(\zeta \to 0\) and \(\text{Re} \to \infty\)) as a compound exterior and interior decision. Euler equation was selected for the exterior solution.

\[
(\nabla \nabla) \overline{w} = -\frac{1}{\rho} \nabla p.
\]

The interior solution can be presented as

\[
u_x + C(x) \nu_{xx} = 0
\]

(11)

where \(u = u(x), u = u(x)\) – certain functions of \(x = \frac{\zeta}{\xi_0}, \xi_0 = O(1/\text{Re}) \to 0\).

Let us write the compound solution for equation (10) [24]

\[
u = u^* + H \exp(-G(x)).
\]

(12)

Here the second term is the solution of the equation (11), \(H\) is a certain constant, \(G(x)\) is the function of \(-x\), \(u^*\) is the exterior solution. Formula (12) is solution of the Navier – Stokes equation (10). The comparison of (12) and (7) shows that these solutions agree to each other at

\[
u_x = \zeta^{1/2}.
\]

In this case \(u_0 = v_0 = 0\) should be assigned on the wall directly.

Thus, solving Euler equations with due regard to (7) it is possible to obtain the result which would not contradict to the solution of (12) where the velocity profile is proportional to \(\zeta^{1/2}\) at \(\zeta \to 0\), \(\overline{w} = 0\) when \(\zeta = 0\). It matches to the facts from [25, 26], wherein the authors show that the profile of tangential velocity near the wall changes by the law of \(\overline{w}_z \sim \zeta^{1/2}\).

**Numerical method and assignment of boundary conditions on the wall**

One way method of finite volume has been realized in the paper presented.

It is advisable, to solve the task from the viewpoint of the community of all possible flows description, including those with gas-dynamic parameters discontinuity, to define a computational scheme based on the approximation of gas dynamics laws written in integral form for each cell of the computational region.

The external flowing around the bodies by a non-viscous, coercible non-thermal-conducting gas is under consideration. The initial equations are Euler equations in integral form. The system of equations is added by Clapeyron equation of state.

The equations are solved in the real space of physical flow (Cartesian system of coordinates is used). Coordinate imagine is not needed, and the simplicity of condition observation on the boundaries remains for any practically essential body configurations.

In addition, this approach has one more advantage – it permits to apply both simple and very general nonorthogonal irregular grids. The method of computational grid creation used in this paper was presented in [27]. The direct use of conservation laws in integral form guarantees
also that the shock waves and other discontinuities of the gas-dynamic parameters will not pre-
vent correct solution, and may be determined without any peculiarity distinguishing.

The flow of gas seems to be unsteady with the assigned initial flow field. The shock-
capturing method is applied.

Consideration is being given to some three-dimension region $G$ with the boundary with humps, in general terms. Region $G$ is divided to cells presenting a form of arbitrary hexahedrons.

In solving complicated gas-dynamic tasks when there is no way to provide the position of discontinuity surfaces, it is advisable to apply shock-capturing numerical schemes.

The scheme is of second approximation order, time-implicit and space explicit. It pos-
sesses weak non-monotony and full conservatism.

The usual kind of boundary conditions is used on all boundaries, except for the solid wall. The conditions of flow symmetry are assigned in the symmetry plane. On eternity the flow pa-
rameters are taken as being equal to free flow parameters. A linear extrapolation is applied on the “output” boundary [28].

According to the calculation scheme accepted, approximation of conditions on the solid wall is performed by introducing a dummy calculation layer. Assignment of second-genus boundary layers is used in the present algorithm [29]. No slip condition is used instead of the boundary slip condition ordinary for the numerical solution of Euler equations.

**Comparison to the experimental data**

To verify the stated approach the comparison has been done to the experimental data from [30]. The subject was the flowing around a flat delta wing with sweep angle of $\chi = 78^\circ$. Considered Mach numbers of free flow were from 2 to 3, attack angles were $\alpha = 14^\circ$ and $18^\circ$.

In these parameter values a separation flow yielding vortexes forms on the wing leeward side. Aside from the head shock wave, inner shock waves form, which indicates the presence of zones with flow supersonic velocity. Figure 1 shows the results on one viewed variant.

Figure 1, $a$ presents streamlines obtained in the calculations with Mach number of 2.75 and angle of attack of $14^\circ$. In the same place there is an example of visualization by laser light-
sheet technique in the experiment of delta wing flowing around (Fig. 1, $b$). The primary and secondary vortex is clearly seen in both cases. But the numerical simulation has allowed to find several details. A tertiary vortex forms in the edge region. Flow topology on the wing leeward side is unsteady in general. Tertiary vortex sizes and configuration vary all the time. It in-
creases, stretches, divides to a series of small vortexes which interact with the primary and sec-
ondary vortexes. This interaction has the effect of small vortexes being taken by the bigger ones, and the tertiary vortex appears again in the edge region. The process repeats. But the sec-
ondary vortex present in the flow constantly also can vary its shape and size. Sometimes it is one vortex, sometimes it occupies the flow region below the primary vortex, stretches and disin-
tegrates to two vortexes. This process depends on the position of the biggest primary vortex. The slow motion of this formation center to the symmetry plane and back to the edge is ob-
served. Figure 1, $c$ shows the streamlines in the edge region in larger scale than in Fig. 1, $a$. This is a time point when the secondary vortex has disintegrated to two, and the tertiary stretched vortex rises from the edge. As for the experimental pattern of flow, such small details of the flow may be hardly registered. If it is so then Wood – Miller classification [30,31] re-
quires further detailing.

The flow pattern is unsteady in general. Certainly the primary and secondary vortexes pre-
sent in each time point but it is also possible to see small unsteady vortexes (see Fig. 1, $a$, $b$), their existence is not presented in the classification [30, 31].
Let our attention be paid on the shock waves position (Fig. 1,b). Let us have this pattern compared to the distribution of conic Mach number obtained numerically (Fig. 1,d). The comparison shows that the numerical simulation allows to define the location of internal waves, but in the regions of supersonic flow on the leeward side which were viewed in the experiments, only high velocity gradients take place in the calculations.

**Conclusion**

The main role in any turbulent flow belongs to large-scale pulsations corresponding to big Reynolds numbers, consequently, small values of viscosity. Hence it follows that for the large-scale motion in the turbulence inertial interval — the major interval in any turbulent flow — gas viscosity is not important. These circumstances make us seek for effective methods of theoretical investigations of vortex flow macro-structures within the framework of a non-viscous gas.

The present paper deals with the simulation of a flow on the delta wing leeward side with high Re numbers for the model of ideal liquid constructed on the base of unsteady Euler equations. The numerical scheme of the calculation for the flowing around the body is organized in such a way to stipulate possible existence of the peculiarities (vortex surfaces, shock waves, etc.). The position of these peculiarities is defined automatically in the course of counting. The major requirement to the system of gas-dynamic equations is the requirement of function boundedness, i.e., impossibility to tend to infinity of velocity, pressure, etc.
The correlation made for the results obtained with the experimental data has demonstrated that the model being suggested describes adequately the macro-structure of vortex flows on the wing leeward side.

REFERENCE